STUDY OF V-SHAPED SPECIMEN CONCENTRATION EFFECT UPON STRESS GRADIENT

Assist. prof. Dr. Dimova B. [1], Assoc. prof. Dr. Vesselinova-Guteva V. [2]
Faculty of Transport – Technical University of Sofia [1], Technical University of Varna [2], Bulgaria

Abstract: *The paper presents the results of numerical and experimental linear elastic analyses carried out to investigate the stress gradient of notched V-shaped specimens. Specimens with different V- shaped notch geometry subjected to uniaxial tensile loading have been considered. It was shown that the stress fields around the notch-tip are similar to each other regardless the notch shape. The most familiar analytical functions describing the stress field at the notch-tip have been presented. Some of the available approximation formulas are verified with the obtained numerical data. Comparison of results is shown and is discussed.*
Keywords: STRESS GRADIENTS, NOTCH-TIP STRESS DISTRIBUTION, FATIGUE, FATIGUE LIFE PREDICTION

1. Itroduction

There is a great variety of approximated and analytical solutions [3, 6, 8] for predicting stress distribution in zones adjoining the notch - tip for the most common concentrators. Despite that these solutions can easily be used for analysis of local stress distribution in real constructions (components with concentrators), the use of numerical methods, FEM in particular, is on the rise notwithstanding the complexity of the analyzed geometries. The processing of the large database derived using FEM is often too labor-consuming and slow compared to the compact and quickly accessible results derived through any analytical method.

Important parameters, which are used to describe the local stress field in proximity to the notch - tip are: stress concentration factor $\mathbf{K_t}$, effective stress concentration factor $\mathbf{K_f}$ and the relative stress gradient χ.

For constructions subject to cyclic loading, it is well known that a fatigue crack may appear all too early in places with local stress concentration and to accelerate its intensive growth in areas close to the notch - tip. After clarifying the opportunities for forecasting durability upon alternating loads [7, 5] and the proposed relation between $\chi = f(K_t/K_f)$ from Neuber [4, 5], a methodology for durability forecasting may be proposed by using the stress gradient. To create such a methodology, it is necessary to know various solutions for its calculation and to determine it for various testing specimens. The relative stress gradient is presented with the following expression

$$\chi_{(\rho_i)} = \frac{1}{\sigma_{max}} \left. \frac{\partial \sigma_y}{\partial x} \right|_{x = x_0}, \qquad (1)$$

where σ_{max} is the maximum whereas $\sigma_y = f(x, d, r, t, \ldots)$ is the normal stress in the notch - tip and is a function of various parameter that depend on the form of the concentrator, the type of load and the formula used to describe the stress field.

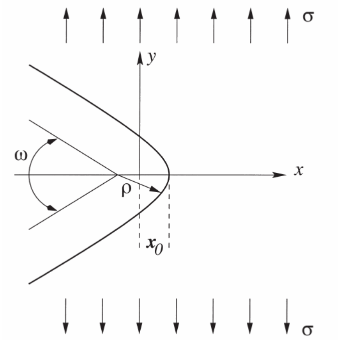
The present paper discusses several basic types of solutions – exact, analytical and approximate, which allow calculate the stress gradient. The results obtained from the reviewed solutions are compared to the results from the numerical modeling of the studied specimens with V-shaped concentrators, varying the form of the specimens (bars and plates) and the geometry of the concentrator upon tensile load.

The data presented here can attend to calculate the stress intensity factors that determine crack growth in the concentrator and to determine fatigue strength of notched elements.

2. Preconditions and means for resolving the problem

2.1 Basic relations

Analyses and studies concerning V-shaped notches have been carried out by various researchers [4, 6, 7, 8]. There exist only a limited number of cases, which can accurately determine an analytical solution of stress distribution for a V-shaped form. They present stress distribution in a zone close to the notch - tip for an infinite plate that is subjected to even tensile loading [4, 5]. Other formulations that describe the stress distribution close to the notch - tip and later reported by Howland and Ling [2] allow calculating the exact stress fields in plates of finite size, whose cross-sections have been enfeebled by similar geometrical concentrators. Such formulations are used more rarely nowadays.



$$\chi = \frac{1 - 2q + s}{q x_0} = (-)\frac{2}{\rho} \qquad (2)$$

$$\chi r_0 = 2$$

Fig.1 *Acute deep V-shaped concentrator of the thin plate subjected to the tension loading and presentation of the analytical solution of Neuber for determination of relative stress gradient for different concentrators.*

Neuber's theory of notched stress concentration [4, 5] is based on the first components of Airy's biharmonic potential function to reduce stress, in particular its maximum value. In use is the condition for having the concentrator radius small compared to the plate width. Then the first components of the developed exponential stress function row are sufficient to describe the stress field around the concentrator, whereas the rest of the components of higher power are ignored. In case the geometric condition is not met, the expressions presented in tables and technical fatigue manuals can be used only as approximated solutions. The expression (2) for the relative stress gradient χ according to equation (1) for stress concentrators is presented in Figure 1. In addition, in this case it is accepted that $\chi r_0 = 2$.

Lately, approximated solutions for the stress field in the notch-tip neighborhood are in growing use. For example, Xu [6] proposed a convenient expression for the calculation of normal stress in the notch - tip by using $\mathbf{K_t}$ and a finite width correction factor $\mathbf{f_w}$. By applying equation (1) and making some substitutions in [6], the relative stress gradient χ may be written in a simplified form, hence the final form of the equation (3).

(3) $$\chi \approx (-)\frac{2.273}{r_0}$$

According to Glinka and Newport [3], normal stress in the notch - tip with $K_t \leq 4.5$ can be presented in a combined equation, which originates from the equation given by Creager and Paris [1, 7] for acute deep notches and that of Usami [1, 7] for elliptical concentrators. To broaden the approach, which has usually been used for cases with deep concentrators or to use the stress field alongside the maximum local stress for predicting the fatigue strength, these authors propose the simplified approximated formula for the stress field of V-shaped concentrators and thereof the equation for the relative stress gradient χ (4).

(4) $$\chi = (-)\frac{2.1667}{r_0}$$

The presented exact, analytical and approximate solutions of Neuber, Xu and Glinka for distribution of the stress field will be taken as the starting point for further studies and comparisons.

2.2 Description of the notched specimens

Figure 2 illustrates schematically the cross-section of the geometry of plate and cylindrical testing specimens with bilateral V-shaped symmetrical concentrators. The stress gradient and the relative stress gradient in the concentrator zone have been studied, as well as some subordinate parameters such as the stress concentration factor. The selection of the stress concentrator shape from the presented geometric models' series takes specimens from previous projects into consideration, and here a new type of V-shaped concentrator form is presented and investigated. The remaining boundary conditions correspond to the already studied and commented in [7, 9] geometries. The geometry models cover the decrease of the concentrator's radius approximation from smooth specimens, through flat concentrators, acute deep notches and such that tend to infinity (i.e. simulate the effect of an existing crack). The numerical experiments are held on specimens with the material characteristics of the magnesium alloy AZ91 upon tensile loading.

Depending on the geometry model of the testing specimens – plate or cylindrical, the notch - tip is subjected to uni-axial or multy-axial stress state conditions.

The series of specimens for calculation have the form exhibited in Figure 2, as follows: plate and cylindrical specimens with bilateral symmetric V-shaped concentrators and various concentrator opening angles α_i [i = 1, 2, 3], where α_1 = 45°, α_2 = 60° and α_3 = 135°. The modeling of all geometry models is coordinated with the trial standard requirements [7]. After the initial studies were prepared and created a total of 24 cylindrical and plate geometry models - 8 for each opening angle.

In the creation of the geometric models, special attention has been paid to having variable parameters alongside constants such as the concentrator parameters so as to allow generalization of the obtained results and their application in the broader-range fatigue calculations.

The radius of curvature in the concentrator tip R_i has been selected as the variable parameter, the radii of curvature being R_i = 0,5 mm, 1 mm, 2,5 mm and 5 mm and the concentrator depth t = 5 mm has been used as the constant parameter. The length and width of the tested specimen are selected as the constant values. The main idea for such geometrical selection and concentrator parameter variation is not to change the area of the cross-section in the concentrator tip so as to facilitate and equalize calculations to find stress concentration factor.

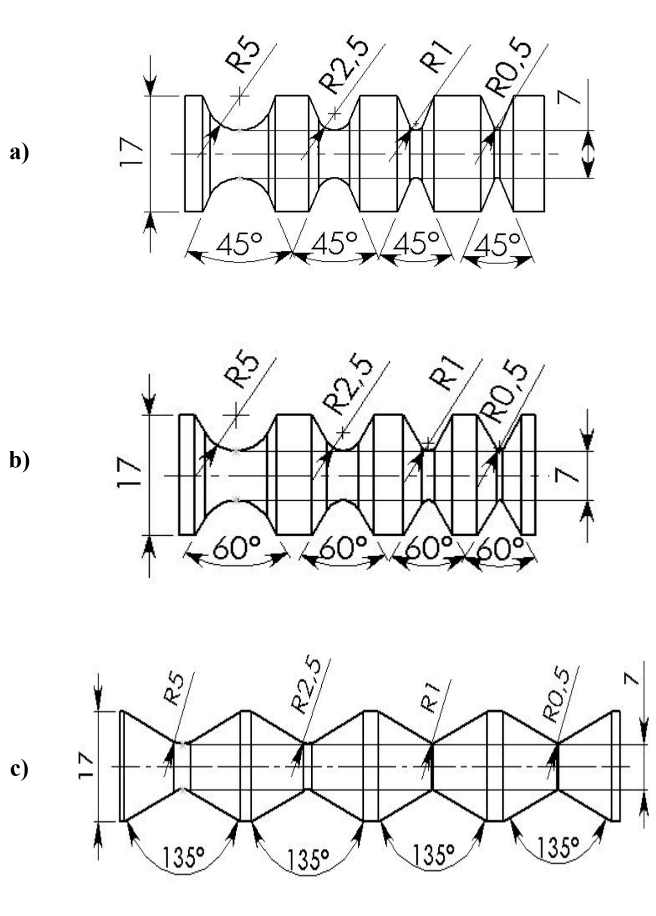


Fig.2 *Schematic presentation of the cross-sectional geometry of the examined notched specimens – rotations bars and plates (thickness d = 6 mm) with bilateral V-shaped symmetrical concentrators used in the analytical, numerical and experimental analyses*
Illustrating the variations of radii of curvature at the notch – tip for various opening angles of the concentrator tip α_i [i = 1, 2, 3]:
a) *α = 45°,* ***b)*** *α = 60°,* ***c)*** *α = 135°*

The obtained numerical and experimental results for different geometries (see Figure 2) form the ***basis for development*** of a simplified approach for numerical prediction of the fatigue life-time of complex structural elements. Computing the stress fields around the notch-tip for examined notched geometries and comparing of the numerically obtained stress fields and those given by the most appropriate formulas gives the necessary insight into research problem.

3. Solution of the investigated problem

The distribution of the stress fields and deformation fields are numerically determined in the notch zone and the stress concentration factors K_t for the various series of specimens and load (tension, bending and torsion) are calculated. The stress concentration factors serve as the basis for obtaining the stress gradient $\partial\sigma_y/\partial x$ and the relative stress gradient χ or for other diagrams that describe the concentrator neighborhood zone.

The numerical solutions data is presented in a processed and generalized data on Figure 3 and Figure 4, where on display are dependencies of a various type, including the stress gradient, the radius of curvature and the depth of the concentrator. Only data about the stress concentration factor K_t on tension is presented since tensile strength is the most dangerous (with the largest stress concentration factor K_t) loading type.

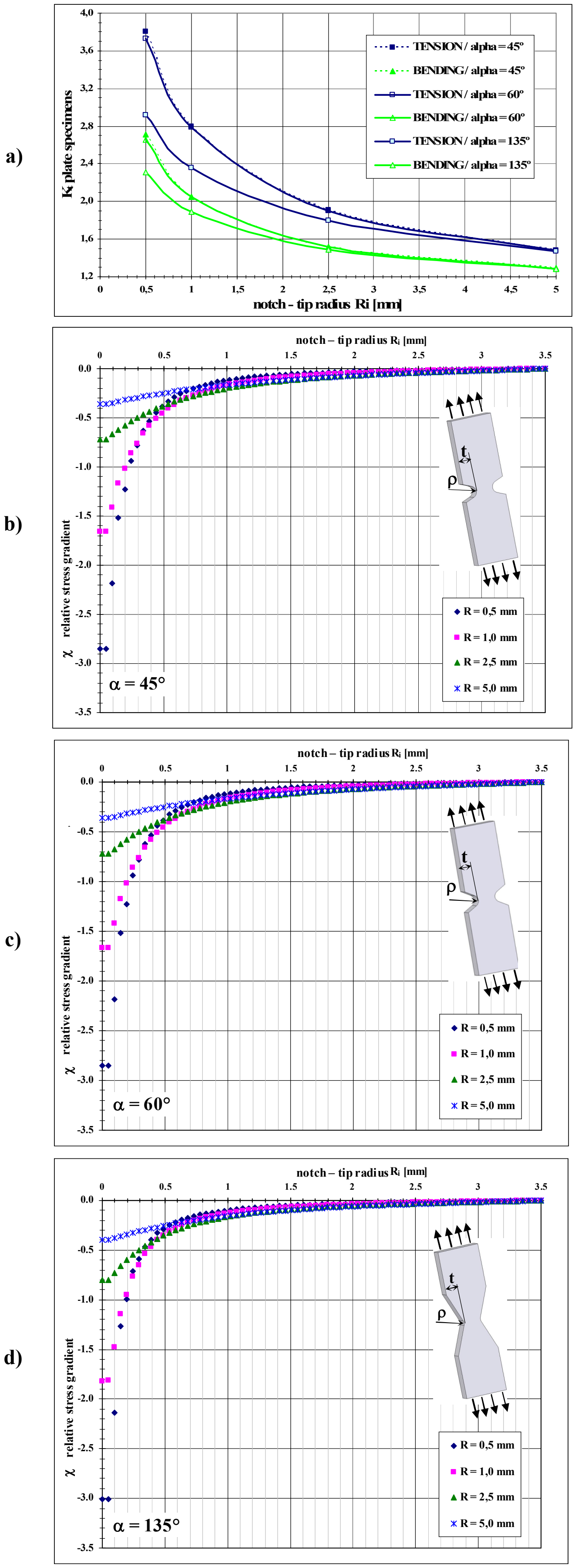


Fig.3 *Basic diagrams for relative stress gradient of a series* ***of plate testing specimens*** *with bilateral V-shaped symmetrical concentrators and various concentrator opening angles* α_i *[i = 1, 2, 3] under tensile loading:*
b) *α = 45°,* ***c)*** *α = 60°,* ***d)*** *α = 135°.*
For all tested specimens is presented also ***a)*** *the stress concentration factor* K_t *for the various series of specimens and loading (tension, bending and torsion).*

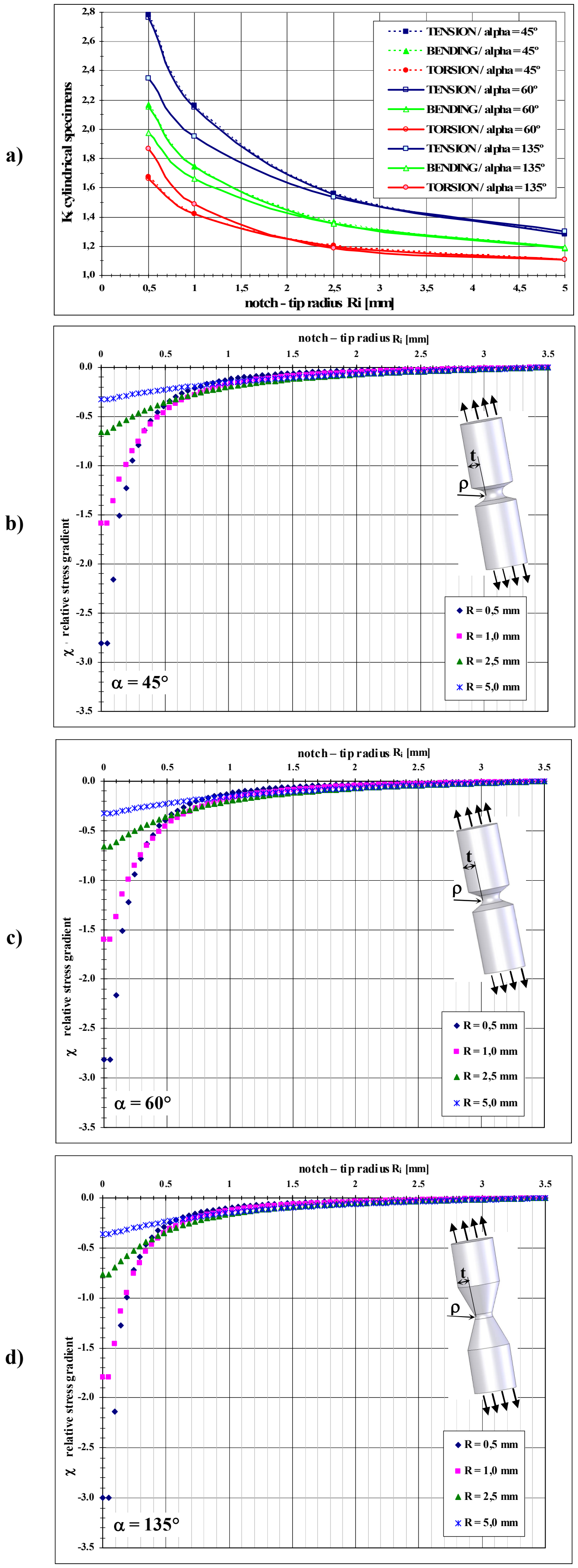
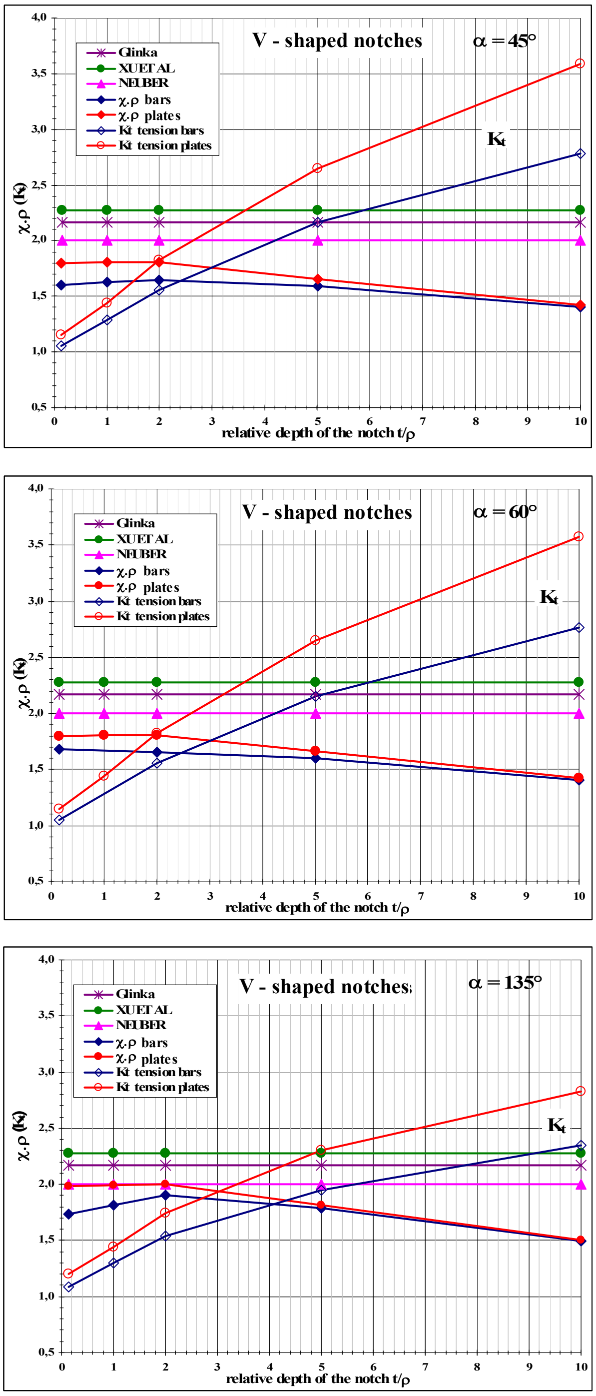


Fig.4 *Basic diagrams for relative stress gradient of a series* ***of cylindrical testing specimens*** *with bilateral V-shaped symmetrical concentrators and various concentrator opening angles* α_i *[i = 1, 2, 3] under tensile loading:*
b) *α = 45°,* ***c)*** *α = 60°,* ***d)*** *α = 135°.*
For all tested specimens is presented also ***a)*** *the stress concentration factor* K_t *for the various series of specimens and loading (tension, bending and torsion).*

4. Results and discussion

Of particular importance is the comparative diagram for $\chi.\rho$, exhibited in Figure 5, which provides an overall outlook on the applicability of the formulations and the results thereof used in the study.



a) b) c)

Fig.5 Comparative diagram for testing specimens with bilateral V-shaped symmetrical concentrators presenting obtaining of the relative stress gradient using different solutions: exact, analytical, semi-analytical, approximate and numerical and various concentrator opening angles under tensile loading: ***a)*** *α = 45°,* ***b)*** *α = 60°,* ***c)*** *α = 135°*
For all tested specimens is presented also the stress concentration factor K_t

It compares and summarizes the analytical expression for $\chi.\rho$ of Neuber, the approximate solutions for $\chi.\rho$ of Hu Et Al and Glinka and the obtained numerical product $\chi.\rho$. What can be observed from Fig. 5 is that the analytical and approximate solutions for the relative stress gradient are not suitable to determine with great precision the distribution of stress in the notch - tip. Relatively close to the numerical results are only the solutions derived for testing specimens with concentrators whose radius or curvature is smaller or equal to 2 ($R_i \leq 2$).

5. Conclusions

The results presented in the presented study allow applying some of the formulations and solutions for the stress gradient and the relative stress gradient. The derived expressions about the relative gradient from Neuber's analytical solution have to be rendered as approximate because they cannot take into account the stress field fading in places far from the concentrator tip. They have to be taken into account only as a border case for a relative stress gradient, which is suitable for elliptical concentrators. The proposed approximate expressions for the relative gradient of Xu and Glinka are also rendered as border cases, which are suitable for concentrators whose radius of curvature is smaller or equal to 2 ($R_i \leq 2$). The results derived for the relative stress gradient with the help of the numerical method give values which are the closest possible to the real values.

The knowledge of the precise local stress fields of the notch - tip allows to correctly present the gradient depending on the stress model but this is possible only for a limited number of geometries. Due to this in a further development an approximate formulation can be sought that describes the stress field in broader terms.

Acknowledgment

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6. Literature

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