Determine the fatigue lifetime for aluminium alloy EN AW 2007.T3 during cyclic bending – torsion loading under in-and-out of phase shift \( \phi = 0^\circ \) and \( \phi = 90^\circ \) using selected fatigue criteria

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Abstract: The article deals with determining of fatigue lifetime of aluminium alloy EN AW 2007.T3 during by multiaxial cyclic loading. The theoretical part deals with the fatigue and with the criteria for evaluation of multiaxial fatigue lifetime, in region low-cycle and high-cycle fatigue. The experimental part deals with modeling of combined bending - torsion loading and determining the number of cycles to fracture in region low-cycle and high-cycle fatigue and also during of loading with the sinusoidal wave form under in phase \( \phi = 0^\circ \) and out phase \( \phi = 90^\circ \).

KEYWORDS: ALUMINIUM ALLOY, FATIGUE CRITERIA, SINUSOIDAL CYCLIC LOADING, MULTIAXIAL FATIGUE, STRESS

1. Introduction

In the Earth's crust, aluminium is the most abundant (8.3% by mass) metallic element and the third most abundant of all elements (after oxygen and silicon). Aluminium is a relatively soft, durable, lightweight, ductile and malleable metal with appearance ranging from silvery to dull gray, depending on the surface roughness. It is nonmagnetic and does not easily ignite. Aluminium is one of the lightest engineering metals, having strength to weight ratio superior to steel. By utilising various combinations of its advantageous properties such as strength, lightness, corrosion resistance, recyclability and formability, aluminium is being employed in an ever-increasing number of applications. This array of products ranges from structural materials through to thin packaging foils [1, 2].

Multiaxial states of stress are very common in structures and components. Fatigue is usually a surface phenomena so that the state of stress is biaxial because the stress normal to a free surface is zero. Another relatively simple combination of different loads is offered by an axle loaded under combined bending and torsion. This loading combination was tested in our and also in many others experiments [3,4]. In spite of this fact, fatigue mechanisms are still not fully understood. This is partly due to the complex geometrical shapes and also complex loadings of engineering components and structures which result in multiaxial cyclic stress-strain states rather than uniaxial.

Multiaxial loading includes states of stress where the individual components of stress and strain can be either in-phase or out-of-phase, sometimes called proportional and nonproportional loading [5,6].

2. Fatigue criteria

Criteria valid for the fatigue lifetime calculation can be classified in three different categories: strain based methods, strain-stress based methods and energy based approaches. There are plenty of hypotheses used for evaluating a degree of damage caused by variable load [7, 8]. Life prediction methods which presume homogeneous material (free from cracks, inclusions or defects) at the outset of the investigation can be divided into strain-based (low-cycle fatigue) and stress-based (high-cycle fatigue) methods. There were chosen some fatigue criteria: Fatemi-Socie, SWT, Brown-Miller, Liu, Goodman, Sines, Findley and MCE fatigue criterion.

Fatemi and Socie [9] observed that the Brown and Miller’s idea could be successfully employed even by using the maximum stress normal to the critical plane, because the growth rate mainly depends on the stress component normal to the fatigue crack. Starting from this assumption, he proposed two different formulations according to the crack growth mechanism: when the crack propagation is mainly MODE I dominated, then the critical plane is the one that experiences the maximum normal stress amplitude and the fatigue lifetime can be calculated by means of the uniaxial Manson-Coffin curve; on the other hand, when the growth is mainly MODE II governed, the critical plane is that of maximum shear stress amplitude and the fatigue life can be estimated by using the torsion Manson-Coffin curve. Criterion has the following form:

\[
\frac{\Delta \varepsilon}{2} \times \left(1 + k \times \frac{\sigma_{\text{max}}}{\sigma_y}\right) = \frac{\tau_f}{G} \times (2 \times N_f) + \varepsilon_f \times (2 \times N_f) \quad (1)
\]

Smith, Watson and Topper (SWT) created a parameter for multiaxial load, which is based on the main deformation range \( \Delta \varepsilon_n \) and maximum stress \( \sigma_{\text{max}} \) to the main plane. Criterion has the following form:

\[
\sigma_{\text{n,max}} \times \frac{\Delta \varepsilon}{2} = \frac{\sigma^2}{E} \times (2 \times N_f)^b + \sigma_f \times (2 \times N_f)^e \quad (2)
\]

Brown and Miller [10] observed that the fatigue life prediction could be performed by considering the strain components normal and tangential to the crack initiation plane. Moreover, the multiaxial fatigue damage depends on the crack growth direction. Different criteria are required if the crack grows on the component surface or inside the material. In the first case they proposed a relationship based on a combined use of a critical plane approach and a modified Manson-Coffin equation, where the critical plane is the one of maximum shear strain amplitude. Criterion, which was created, has the following form:

\[
\frac{\Delta \varepsilon_{\text{max}}}{2} + S \times \Delta \varepsilon_n = A \times \frac{\sigma_f^2}{E} \times (2 \times N_f)^b + B \times \varepsilon_f \times (2 \times N_f) \quad (3)
\]
Liu created a virtual model of the deformation energy, which is a generalization of the axial energy on the basis of prediction of fatigue life. Criterion has the following form:

$$\Delta W = 4 \times \sigma_y \times \varepsilon_y \times (2 \times N_y)^{k+e} + \frac{A \times \sigma^2}{E} \times (2 \times N_y)^{2b}$$  \hspace{1cm} (4)$$

Goodman used main stresses for evaluating the fatigue under multiaxial loading. Normal stresses are calculated for each plane and their ranges are used for calculation of fatigue lifetime. If the point of the combined stress is below the relevant Goodman line then the component will not fail. This is a less conservative criteria based on the material ultimate strength yield point $S_{ut}$. To establish the factor of safety relative to the Goodman’s criteria can be written as:

$$\frac{K_f \times \sigma_{amp}}{S_e} + \frac{\sigma_{mean}}{S_{ut}} = \frac{1}{f_f}$$ \hspace{1cm} (5)$$

Sines published his works throughout the fifties of the last century. His criteria are very much alike, utilizing the amplitude of second invariant of stress tensor deviator (which corresponds to the von Mises stress) as the basis. Another term is added to the equation in order to cope with the mean stress effect – while Sines prefers the mean value of first invariant of stress tensor (i.e. hydrostatic stress $\sigma_0$). His resulting failure criterion can be expressed as:

$$\frac{\Delta \tau_{oct}}{2} + a \times (3 \times \sigma^{'\text{mean}}) = \tau_f \times (N_f)^b$$ \hspace{1cm} (6)$$

Findley criterion is the first critical plane criterion. He suggested that the normal stress $\sigma_n$, acting on a shear plane might have a different linear influence on the allowable alternating shear stress, $\Delta \tau/2$. Criterion has the following form:

$$\frac{\Delta \tau}{2} + k \times \sigma_n = \tau_f \times (N_f)^b$$ \hspace{1cm} (7)$$

Minimum circumscribed ellipse (MCE) – The origin of this method goes out from minimum circumscribed circle method (MCCM) [11]. This method was first presented by Papadopoulos. Its major feature is its explicitness in determination of mean shear stress. Papadopoulos later shows that such minimum circumscribed circle can be obtained by a search through all pairs and triads of points in the shear stress path, but such an approach can be very lengthy. The contrast in comparison with MCCM is clear – it should offer a better solution of phase shift effect problems. Nevertheless, as regards the definition of mean shear stress, it does not offer any new approach. For proportional loading this will always be a straight line and for non-proportional loading histories will have some complex shape.

$$\tau_n = \sqrt{R_y^2 + R_z^2}$$ \hspace{1cm} (8)$$

Where: $\gamma_f$ is the fatigue ductility coefficient in torsion; $\sigma_f$ is the fatigue ductility coefficient; $\sigma_y$ is the fatigue strength coefficient; $\sigma_0$ is the mean hydrostatic stress; $\sigma_n$ is the normal stress; $\sigma_{\text{amp}}$ is the maximum stress; $\sigma_{\text{mean}}$ is the mean stress; $\varepsilon_y$ is the stress in the direction of the axis y; $\tau_n$ is the equivalent shear stress; $\tau_f$ is the fatigue strength coefficient in torsion; $\Delta \tau_{\text{max}}$ is the maximum shear strain range; $\Delta \varepsilon_{t}$ is the principal strain range; $\Delta \varepsilon_{n}$ is the normal strain range; $\Delta \tau_{oct}$ is the alternating shear stress; $\Delta \tau_{\text{oct}}$ is the octahedral shear stress; $\Delta W$ is the virtual strain energy; $N_f$ is the number of cycles to fracture; $S_{ut}$ is the modified fatigue strength; $S_{ut}$ is the ultimate tensile strength; $f_f$ is the factor of safety applicable the fatigue; $E$ is the elasticity modulus in tension; $G$ is the elasticity modulus in torsion; $R_y$ is the major axis of the ellipse; $R_z$ is the maximum distance of stress point; $b$ is the fatigue strength exponent; $c$ is the fatigue ductility exponent; $\alpha$ is the fatigue ductility exponent in torsion; $A, B, S, k, a$ are material parameters.

3. Numerical calculations and results

In ANSYS software was created the model of the test bar. The real geometry of this component is shown in Fig.1. The rod bar had a circular shape with a defined section, in which was expected an increased concentration of stress and creation a fatigue fracture.

![Fig.1 Geometry of the test bar](image)

The ends of this model were loaded by reversed bending moment on the one side and by reversed torsion moment on the opposite site. The values of presented stresses and strains in the middle of the rod radius were taken from computational analysis using finite element method. We used the following parameters in finite element model: used material was aluminum alloy EN AW 2007.T3 (AlCu4PhMg) with Young's modulus $E = 0.817 \times 10^{11}$ Pa, Poisson number $\mu = 0.3$ and with the strength limit $R_{\text{ut}} = 491$ MPa. From computational analysis can be seen that the area with greatest concentration of stresses or eventually the place with the higher deformation was localized in the middle of the rod radius (see Fig.2).

![Fig.2 Result of FEM analysis in ANSYS software](image)
Obtained values of the stresses from finite element analysis were next computational analyzed using Fatigue Calculator software. This is a program which can quickly calculate fatigue lifetime of selected material. After starting the calculation, Fatigue Calculator displayed the number of cycles to failure for different models of damage. In our calculation we considered with all multiaxial criteria described above which can be applied to low-cycle and also to high-cycle fatigue region. All the tests were performed under controlled bending and torsion moments. Frequency of each analysis was equal to 30 Hz. It was first detected the number of cycles to fracture for multiaxial low-cycle fatigue with amplitudes in the phase shift 0° and then out of the phase shift 90° for stress. The same was done for multiaxial high-cycle fatigue. The obtained numbers of cycles are processed into Wöhler curves $\sigma_{xx} - \log N_f$ for multiaxial cyclic combined bending - torsion loading. For multiaxial low-cycle fatigue with phase shift 0°, Wöhler curves are shown in Fig.3. For multiaxial low-cycle fatigue with phase shift 90°, Wöhler curves are shown in Fig.4.

4. Conclusion

All multiaxial models applied to fatigue lifetime calculation of aluminum alloy EN AW 2007.T3 increases with decreasing stress amplitude continuously in the cycles of number region. Comparing Wöhler curves for low-cycle fatigue (see Fig.7), for amplitudes of the load with phase shift 0° (solid lines) and for amplitudes of the load with phase shift of 90° (blank lines), it can be seen that some models (such as Fatemi-Socie and SWT) give higher resistance to fatigue damage in the phase shift than the synchronized load amplitudes. This may be caused by, that the bending loading and neither torsion loading not active with the maximum value on the sample at the same time during the phase shift, but alternately. In this way, as if the sample was loaded by lower value of stress or deformation in a given time (phase shift of 90°). For other models, this shift of amplitudes did not cause any significant changes and the differences are minimal.
Fig. 7 Comparison of Wöhler curves for multiaxial low-cycle fatigue

Comparing Wöhler curves for high-cycle fatigue (see Fig. 8), for amplitudes of the load with phase shift 0° (solid lines) and for amplitudes of the load with phase shift of 90° (blank lines), it can be seen that all models (except for Sines) gives a higher resistance against fatigue damage in the phase shift than in the synchronized amplitudes of loading. Probably the reason will be same as for low-cycle fatigue.

Fig. 8 Comparison of Wöhler curves for multiaxial low-cycle fatigue

It was observed that a phase shift 90° is the cause of "rotating" curves of fatigue life, which may have an impact on partial increase of fatigue life for the area of low-cycle and high-cycle fatigue.

5. Acknowledgements

This work has been supported by Scientific Grant Agency of Ministry of Education of Slovak Republic and Slovak Academy of Science, No.1/0683/15.

6. References

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