Abstract: In this paper is presented Synthesis of a four-bar linkage mechanism of the lift car extrusion. In this mechanism are introduced even higher kinematic pairs. The movement of the mechanism is repeated periodically and it is sufficient to do its kinematic study for an angle 75 [deg]. The description of the mechanism movement can be performed in the grafonoanalytical or analytical path by centers of speed moments, which belong to a narrow link and a loop of the center mechanism, and the instant centers belonging to the two related movements. In the kinematic analysis of the forward mechanisms are used graphical methods. These are simple and universal, making it possible to determine the positions, velocity and acceleration of the links of any structure. With the application of contemporary calculating technology, the graphical methods in the analysis of mechanisms take the right place. The velocity of each link of the mechanism linkage that performs the movement of the plane can be shown as very geometric of the instant center speed and the speed of rotation around the center of the instantaneous. The analysis will be performed by Math Cad software, while kinetostatic analysis will be carried out using Contour Method, comparing results of two different software’s Math CAD and Working Model. The simulation parameters will be computed for all points of the contours of mechanism. For the simulations results we have use MathCad and Working Model software’s.

Keywords: MECHANISM; FOLLOWER; TRAJECTORY; DESIGN; VELOCITY; ACCELERATION; PROGRAM

1. Introduction

Kinematics and dynamic study of mechanisms using plans and graphic methods is known that permissive error. To avert this error is required use of analytic methods, especially for dynamic study of smart mechanisms, where error is not allowed. In despite of hard work during calculations which are required for analytic study, for smart mechanisms this method is required

This paper has been realised with two application software’s: Math Cad and Working Model. In this paper is presented a five-bar linkage ACDH and a simple crank mechanism GEE as shown in the figure below. In this mechanism firstly is determined \( F_X, F_Y \) and \( \theta \). The masses are adopted, since the moments of inertia need to calculate. The kinematic part of the paper will be completed by finding the velocities and accelerations of each point A, B, C, D, E, H and G. In this way are determined the angular accelerations and velocities of the linkages 2, 3, 4 and 5. Whereas for the kinetostatic part will be determined the reaction forces of the points: \( X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, N = X_F \) and \( F_E \) which are acting on the leading link 2. In the picture are shown 5 bodies: \( AB, BC \), right triangle, rod EF, slider bar F.

![Fig. 1: Linkage ADH, and crank mechanism ACH](image)

\[
\begin{align*}
w &= 3n - 3p_5 - p_4 \\
n &= \frac{n - p_5(1 - 0)}{2} \\
p_5 &= 7 \\
p_4 &= 0 \\
W &= 3n - 2p_5 - p_4 \\
W &= 1 \\
r_3 &= 10, r_4 = 10, 5 \leq \theta_3 \leq 70[\text{deg}] \\
HB &= \frac{r_3}{2}, BE = 1, HG = 1 \\
x_E(\theta) &= (HB - BE) \cos(\theta_3), y_E(\theta) = (HB + BE) \sin(\theta_3) \\
x_D(\theta) &= r_3 \cos(\theta_3), y_D(\theta) = r_3 \sin(\theta_3) \\
DE(\theta) &= \sqrt{(y_D(\theta) - y_E(\theta))^2 + (x_D(\theta) - x_E(\theta))^2} \\
\alpha(\theta) &= \frac{(HB + BE) \sin(\theta)}{(HB - BE) \cos(\theta)} \\
\beta &= \frac{\sin(\theta_3) - (HB + BE) \sin(\theta)}{r_3 \cos(\theta_3) - (HB - BE) \cos(\theta)} \\
\gamma &= 90\text{deg} + \beta(\theta_3) - \alpha(\theta_3) \\
v_E &= \frac{v}{\cos(\gamma(\theta_3))} \\
\omega_2 &= \frac{v}{DE(\theta)} \\
\omega_2 &= 1.8[\text{deg}] \\
\end{align*}
\]
\[ \beta = a \tan \left[ \frac{r_3 \sin(\theta_3) - (HB + BE) \sin(\theta_3)}{r_3 \cos(\theta_3) - (HB - BE) \cos(\theta_3)} \right] \]

\[ \gamma = 90 \text{deg} + \beta(\theta_3) - \alpha(\theta_3) \]

\[ v_E = \frac{v}{\cos(\gamma(\theta_3))} \]

\[ \omega_2 = \frac{v_E(\theta_3)}{DE(\theta_3)} \]

\[ \omega_2 = 1.81[\text{deg}] \]

\[ \theta_3 = 0 \]

\[ \frac{d}{dt} \theta_3 = t \]

\[ \frac{d}{dt} v_E(\theta_3) = \omega_2(t) \]

\[ \alpha(t) = a \tan \left[ \frac{(HB + BE) \sin(\theta_3(t))}{(HB - BE) \cos(\theta_3(t)) + HG} \right] \]

\[ \theta_3(0.5) = 0.425 \]

\[ v_{E1}(t) = \sqrt{\left( \frac{d}{d\theta_3} x_E(\theta_3) \right)^2 + \left( \frac{d}{d\theta_3} y_E(\theta_3) \right)^2} \]

\[ \omega_{3a}(t) = \frac{d}{dt} \omega_3(t) \]

\[ v_{E1}(2.21) = 7.068 \]

\[ a_{E1}(t) = \frac{d}{dt} \left( \frac{d}{d\theta_3} x_E(\theta_3) \omega_3(t) \right) = \frac{d^2}{d\theta_3^2} x_E(\theta_3) \omega_3(t) \]

\[ v_{E5}(t) = \frac{d}{dt} y(\theta_3) \omega_3(t) \]

\[ a_{E5}(t) = \frac{d^2}{d\theta_3^2} y(\theta_3) \omega_3(t)^2 + \frac{d^2}{d\theta_3^2} y_E(\theta_3) \epsilon_3(t) \]

\[ a_{E1}(0.5) = -0.806 \]

\[ a_{E5}(0.5) = -1.71 \]

\[ a_{E1}(t) = \sqrt{a_{E1}(t)^2 + a_{E5}(t)^2} \]

\[ a_{E1}(0.5) = 1.891 \]

Point A

\[ x_A(\theta_3) = r_3 \sin(\theta_3), y_A(\theta_3) = 0 \]

\[ v_{A1}(t) = \frac{d}{d\theta_3} x_A(\theta_3) \omega_3(t), v_{A5}(t) = 0 \]

\[ v_{A}(t) = \sqrt{v_{A1}(t)^2 + v_{A5}(t)^2} \]

\[ a_{A}(t) = 0 \]

\[ a_{A}(0.5) = 2.85 \]

Point B

\[ x_B(\theta_3) = \frac{r_3}{2} \cos(\theta_3), y_B(\theta_3) = \frac{r_3}{2} \sin(\theta_3) \]

\[ v_{B1}(t) = \frac{d}{d\theta_3} x_B(\theta_3) \omega_3(t), v_{B5}(t) = \frac{d}{d\theta_3} y_B(\theta_3) \omega_3(t) \]

\[ v_B(t) = \sqrt{v_{B1}(t)^2 + v_{B5}(t)^2}, v_B(0.5) = 2.245 \]

\[ a_{B1}(t) = \frac{d^2}{d\theta_3^2} x_B(\theta_3) \omega_3(t)^2 + \frac{d^2}{d\theta_3^2} y_B(\theta_3) \epsilon_3(t) \]

\[ a_{B5}(t) = \frac{d^2}{d\theta_3^2} y_B(\theta_3) \omega_3(t)^2 + \frac{d^2}{d\theta_3^2} y_B(\theta_3) \epsilon_3(t) \]

\[ a_B(t) = \sqrt{a_{B1}(t)^2 + a_{B5}(t)^2} \]
2. Kinetostatic analysis of the linkage mechanism

Given data:

\[ \begin{align*}
    HG &= 1, \quad m_3 = 10, \quad m_2 = 10, \quad m_4 = 50, \\
    \theta &= 5 \text{ deg}, 5.5 \text{ deg}, \ldots, 70 \text{ deg}, \\
    J_{B_3} &= 83.34, \quad J_{B_4} = 83.34, \quad g = 9.80, \quad t = 0.001 \ldots 2.21 \\
    F &= 0 \\
    F_x(t) &= F \cos(\alpha(t)), \quad F_y(t) = F \sin(\alpha(t)) \\
    x_1(t) &= BE \cos(\theta_3(t)), \quad y_1(t) = BE \sin(\theta_3(t)) \\
    X_B &= 0, \quad Y_B = 0, \quad X_H = 0, \quad X_D = 0, \quad Y_D = 0 \\
    F_A &= 0, \quad F_C = 0
\end{align*} \]

Linkage I

The equilibrium conditions for the point B are equal to zero. Six linkages are used for the kinetostatic analysis. For the first linkage are given the following equilibrium conditions \( X_B, Y_B \) and \( X_D, Y_D \) for the middle points of the linkage BD, point C2 and for the body mass \( M_2 \).
Dynamic equations of motion for link 1:
\[ m_2a_{B_0}(t) = -\alpha_B + X_D - F_x(t) \]
\[ m_2a_{B_0}(t) = F_A + Y_B - F_y(t) + Y_D \]
\[ J_{B_0}\dot{\theta}_3(t) = F_A \frac{r_3}{2} \cos(\theta_3(t)) + Y_B \frac{r_2}{2} \sin(\theta_3(t)) - X_D \frac{r_2}{2} \alpha \]
\[- (F \cos(\alpha(t)))y_1(t) - (F \sin(\alpha(t)))x_1(t) \]

**Linkage II**
Also, for the linkage II are written the equilibrium conditions BC, which are \( X_B, Y_B, X_D, Y_D \) center \( B_3 \) body mass \( F_H \) and moment of inertia for the point \( B_3 \).

Dynamic equations of motion for link 2:
\[ m_3a_{B_0}(t) = X_H + X_E \]
\[ m_3a_{B_0}(t) = Y_H + Y_B + F_C \]
\[ J_{B_0}\dot{\theta}_3(t) = X_H \frac{r_3}{2} \sin(\theta_3(t)) + Y_H \frac{r_3}{2} \cos(\theta_3(t)) - 
- F_C \frac{r_3}{2} \cos(\theta_3(t)) \]

**Linkage III**
Also, for the linkage III are written the equilibrium conditions FD, which are \( X_F, Y_F, X_D, Y_D \) center \( M_4 \) body mass \( F_D \) and moment of inertia for the point \( B_3 \).

Dynamic equations of motion for link 3:
\[ 0 = X_D \]
\[ m_4a_{D_0}(t) = -m_4g - Y_D - F_C \]
\[ 0 = F_C \frac{r_3}{2} \cos(5\,\text{deg}) - Y_D \frac{r_3}{2} \cos(5\,\text{deg}) \]
In previous equations are 9 unknown forces-moments:
\[ Z_{g_i}(t) = \text{Find}(X_B, Y_B, X_D, Y_D, X_H, Y_H, F_A, F_C, F) \]
\[ X_B(t) = Z_{g_i}(t) \]
\[ Y_B(t) = Z_{g_i}(t) \]
\[ X_D(t) = Z_{g_i}(t) \]
\[ Y_D(t) = Z_{g_i}(t) \]
\[ F_A(t) = Z_{g_i}(t) \]
\[ F_C(t) = Z_{g_i}(t) \]
Where: \( Xi \) and \( Yi \) are components of joint forces

![Fig. 5 Linkage I–Equilibrium conditions for the point AD](image1)

![Fig. 6. Linkage I–Equilibrium conditions for the point CH](image2)

![Fig. 7. Linkage I–Equilibrium conditions for the point CD](image3)
Results for 15 positions:
Results for each point are given below where step varies by $\Delta t$

\[
R_B(t) = \sqrt{(X_B(t))^2 + (Y_B(t))^2}
\]

\[
R_D(t) = \sqrt{(X_D(t))^2 + (Y_D(t))^2}
\]

\[
R_H(t) = \sqrt{(X_H(t))^2 + (Y_H(t))^2}
\]

\[
R_A(t) = F_A(t) - R_B(t)
\]

\[
R_C(t) = F_C(t)
\]
Conclusions and recommendations

In this paper are made the calculations of all positions (displacement) for the whole mechanism, and also are determined the plans for velocities and accelerations for each point.

However, in this paper are shown the outline planes of the mechanism same as the diagrams for each linkage through Math Cad software, Six-bar linkage mechanism diagrams which are derived by Working Model are almost similar to the diagrams derived by Math Cad, Through Working Model software, are derived the results of reactions from the equilibrium conditions of six bar linkage mechanism, angular velocities and accelerations for the points A, B and C, for the angles $\theta_1$, $\theta_3$ and $\theta_5$ in time domain through the command Run, As the general conclusion; the results derived by both software, same as for their diagrams, for all points of the six bar linkage mechanism are within the reasonable boundaries.

2. References

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3. Simulation for mechanism by Working Model software

In the second part of this paper is carried out the simulation for all points A,C,D,H and G (time dependent) of linkage mechanism by Working Model, which is shown in the following.

Fig. 8: Diagrams in Working Model for angular velocity and acceleration 3, 4 and 5