COMPARATIVE ANALYSIS OF STATIC AND DYNAMIC ELASTIC MODULUS OF POLYMER CONCRETE COMPOSITES

СРАВНИТЕЛЕН АНАЛИЗ ЗА СТАТИЧНИЯ И ДИНАМИЧНИЯ МОДУЛ НА ЕЛАСТИЧНОСТ ПРИ ПОЛИМЕРБЕТОННИ КОМПОЗИТИ

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Abstract: *The article presents a comparative analysis of the quantitative values obtained from the experimental tests, of the static and dynamic modules of elasticity (Young's modulus) of gamma polymer-concrete composites. The same will be used as constructive material for parts and body elements. Standard test methods are applied. The values of the elastic parameters of the tested polymer-concrete composites are necessary as input data for the various engineering analysis softwares.*

Keywords: *POLYMER CONCRETE COMPOSITES, DYNAMIC ELASTIC PROPERTIES, COMPARATIVE ANALYSIS, DYNAMIC MODULES.*

1. Introduction

For the quantitative assessment of the elastic properties of the actual materials, the so called elastic properties are applied, which are determined by experiment. They are of important practical significance for calculating the elements and structures of strength and hardness, by the various software products for engineering analysis.

The elastic properties include the elasticity modulus E (Young's modulus), the modulus of tangential elastic deformation G (Culon modulus), Poisson's ratio μ, and the modulus of comprehensive pressure K. They are interrelated; where two of them are independent, and thus assumed as basic. Most often in practice, the elasticity modulus E and the shear modulus G are determined experimentally, while Poisson's ratio μ and the modulus of comprehensive pressure K are calculated on their basis, by well known mechanical formulae [1].

The elastic properties have a certain physical meaning, and for the traditional materials they are assumed as constants. For polymers and polymer composites it is found experimentally that the values of the elastic properties determined in the cases of static loading differ considerably from the values determined by the dynamic methods. This difference for some composites can be significant, [5].

Subject of the study of this article are 15 thermoreactive, quasi-isotropic, viscoelastic polyester polymer concrete composites, made in the Laboratory for testing and studying metal-cutting machines, at Technical University of Sofia, Plovdiv Branch [6]. The same will be used as structural material for bodies and body parts of production machines.

Subject of study of this work are: the elastic properties of the range of polymer concrete composites.

2. Theoretic prerequisites

It is well known the method for determining experimentally the static Young's modulus on the basis of the standard ASTM C 580 - 02 (2012), [4]. The core of the method consists of testing by three-point bending of the experimental samples, where the cross sections of the sample "girder" are characterized by normal tensions of the bending moment, Fig. 1. Loading the samples with focused static force, we measure the sagging f at the loading point.

Within the limits of proportionality, where Hooke's law is valid, we can properly write the following:

$$f = \frac{Pl^3}{48EJ_y} \quad (1)$$

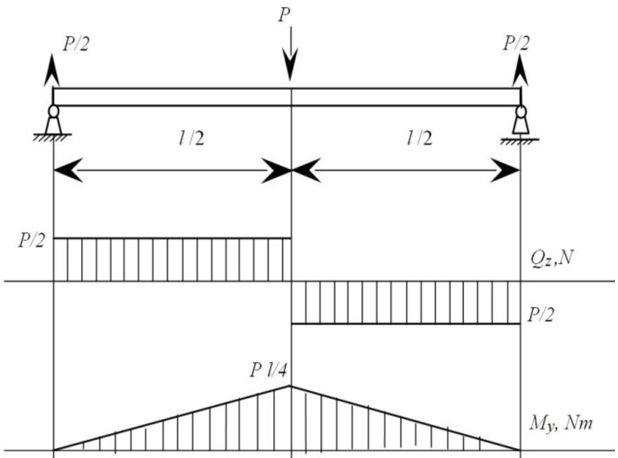

***Fig. 1**. Diagrams for bending*

where:

P - is a focused static force;

EJ_y - hardness of bending;

E – linear deformation modulus;

J_y - moment of inertia versus the central inertia axis y;

l - distance between the supports.

From (1) we express the force P :

$$P = \frac{48EJ_y}{l^3}.f \ , \quad (2)$$

The inertia moment for rectangular section is:

$$J_y = \frac{b.h^3}{12} \quad (3)$$

where:

b - is the width of the sample section;

h - height of the sample section.

We can write the following after substituting of (3) in (1) and (2) :

$$f = \frac{Pl^3}{4Ebh^3} \quad (4)$$

$$P = \frac{4Ebh^3}{l^3}.f \quad (5)$$

The functional relation between the loading force P and sagging (deformation) f of the sample body looks as shown on Fig. 2.

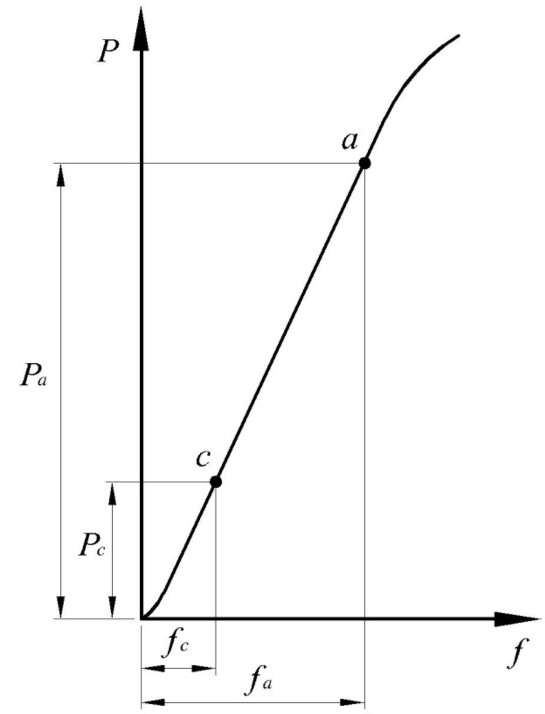

Fig. 2. *Functional relation between P and f*

For the points *a* and *c* within the proportionality limits of the dependence of *P* from *f*, we can write the following:

$$P_a = \frac{4Ebh^3}{l^3} . f_a \ , \qquad (6)$$

$$P_c = \frac{4Ebh^3}{l^3} . f_c \qquad (7)$$

$$P_a - P_c = \frac{4Ebh^3}{l^3} . \, (f_a - f_c) \qquad (8)$$

From (8) we draw the dependency relation for Young's static modulus – E:

$$E = \frac{(P_a - P_c) l^3}{4bh^3 (f_a - f_c)} \qquad (9)$$

The dynamic elastic modulus is determined on the basis of the standard ASTM E1876 - 09 [3] by the equation:

$$E_d = 0.9465 (m f_f^2 / b)(L^3 / t^3) T_1 \qquad (10)$$

where:

E_d - is Young's dynamic modulus;

m - girder's weight;

b - width of the girder;

L - length of the girder;

t - thickness of the girder;

f_f - fundamental resonance frequency at bending;

T_1 - geometric correction coefficient.

The fundamental resonance frequency at bending is measured according to standard. [3]. This is shown on Fig.3

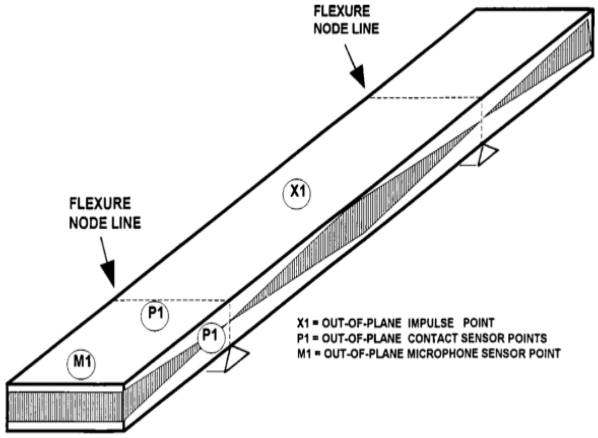

Fig. 3. *Setting for measuring the vibrations at bending*

The geometric correction coefficient T_1 we determine depending on the ratio L/t, as follows:

L/t > 20 - $T_1 = [1.000 + 6.585 (t/L)^2]$ (11)

L/t < 20, then the value of T_1 is calculated by:

$$T_1 = 1.000 + 6.585(1 + 0.0752\mu + 0.8109\mu^2)(t/L)^2 - 0.868(t/L)^4 - \\ - \frac{8.340(1 + 0.2023\mu + 2.173\mu^2)(t/L)^4}{1.000 + 6338(1 + 0.1408\mu + 1.536\mu^2)(t/L)^2} \qquad (12)$$

If Poisson's ratio μ is unknown, we have to assume its initial hypothetical value μ_0. Subsequently, it can be determined by an iteration procedure based on calculation of the dynamic shear modulus G_d, according to the algorithm shown on Fig.4.

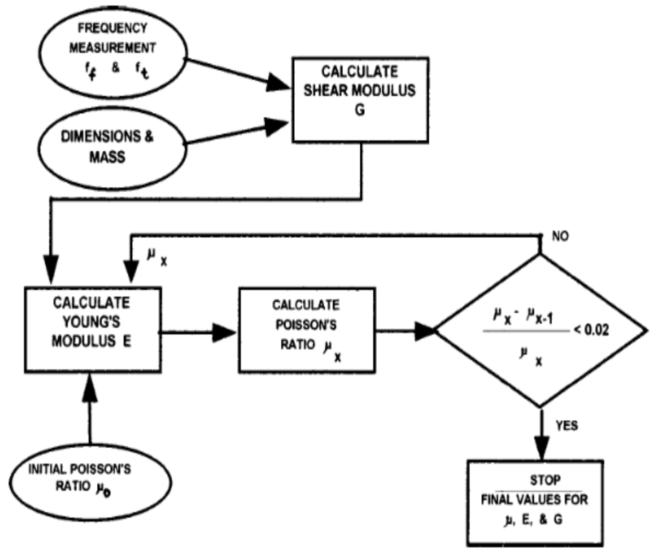

Fig. 4. *Algorithm for determining Poisson's ratio*

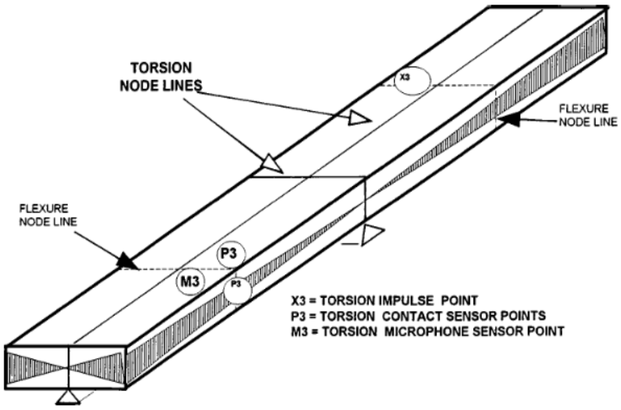

Fig. 5. *Measurement of the vibrations of torque*

The dynamic shear modulus G_d is calculated by the following equation:

$$G_d = 4(Lmf_T^2 / bt)[B/(1+A)] \qquad (14)$$

where:

f_T - is the fundamental resonance frequency at torque. It is measured according to the standard [3]. This is shown on Fig. 5.

A and *B* are correction empiric coefficients for width and thickness.

$$B = \frac{b/t + t/b}{t/(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6} \qquad (15)$$

$$A = \frac{[0.5062 - 0.8776(b/t) + 0.3504(b/t)^2 - 0.0078(b/t)^3]}{[12.03(b/t) + 9.892(b/t)^2} \qquad (16)$$

In the case of isotropic behavior of the material, Poisson's ratio is calculated, by the relation:

$$\mu = (E_d / 2G_d) - 1 \qquad (17)$$

Fig.6. *Experimental setting for determining the static modulus*

3. Results from the experiments

The experimental samples are with the shape of a rectangular parallelepiped (girder type) with dimensions 30x30x350 mm, in compliance with the common standardized norms. The number of polymer concrete composites is 15, of which a total of 45 bodies are made (3 pieces of each composite).

The stand for determining the static modulus of linear deformation E at bending is shown on Fig. 6. The experimental setting for determining the dynamic modulus E_d is shown on Fig. 7. The frequencies f_f and f_T are determined by the frequency ranges, obtained from impulse excitation of the samples, Fig.8 and 9.

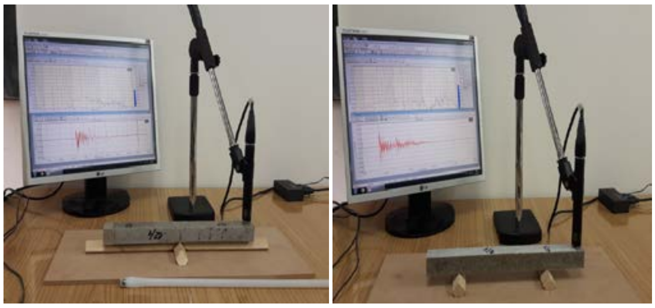

Fig. 7. *Experimental setting for determining* f_f *and* f_T

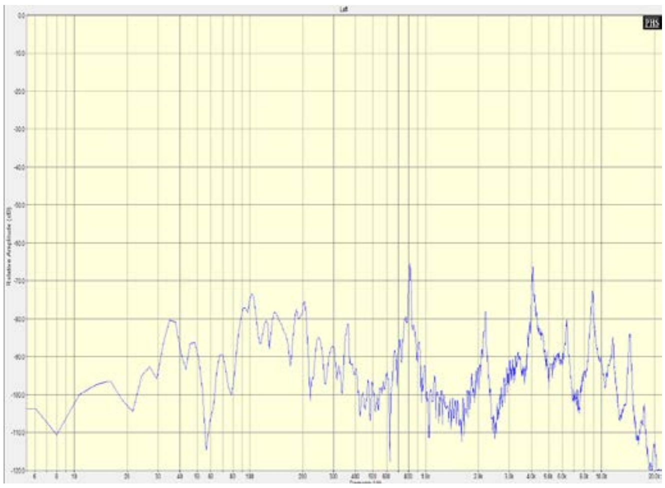

Fig. 8. *Frequency range for* f_f *sample 3.2*

Fig. 9. *Frequency range for* f_T *sample 3.2*

The obtained experimental results for the average values of the modules of linear deformation for each experimental polymer concrete composite, after calculation, are presented in Table 1.

Average module values ***Table 1***

sample №	*E [Pa]*	E_d *[Pa]*	relative difference %
1	23,9E+9	33,7E+9	29,05%
2	19,2E+9	28,9E+9	33,58%
3	36,8E+9	42,3E+9	13,09%
4	19,5E+9	29,0E+9	32,93%
5	16,7E+9	25,0E+9	33,26%
6	12,2E+9	19,3E+9	36,50%
7	15,1E+9	23,7E+9	36,25%
8	14,1E+9	22,3E+9	36,86%
9	17,4E+9	24,5E+9	28,84%
10	16,2E+9	26,7E+9	39,20%
11	15,8E+9	24,4E+9	35,26%
12	13,8E+9	22,1E+9	37,60%
13	18,3E+9	27,8E+9	33,96%
14	25,8E+9	34,3E+9	24,72%
15	16,3E+9	24,3E+9	32,88%

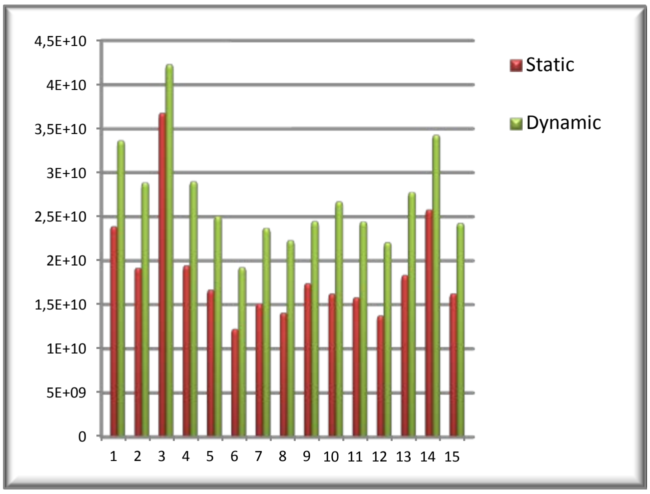

Fig. 9. *Histogram of the static and dynamic modules*

The results of the articles may be summarized as follows:

The quantitative values of the elasticity modules of 15 different polymer concrete composites are obtained. In determining the static modulus E by the method of three-point bending, the saggings f_i and loadings P_i are measured experimentally.

In determining the dynamic modulus, the fundamental bending f_f and torque f_T frequencies of the tested samples were measured with the help of the experimental modal analysis, within their frequency ranges.

A comparative analysis was carried out of the values of the static and dynamic composite elasticity modules. The composite with maximum values of the modulus was determined.

The opportunities for obtaining credible information about the modules for this type of composites, on the basis of the proposed methodology and measurement equipment, are real and adequate.

4. Conclusions

Analyzing the obtained values of the static and dynamic Young’s modules, we reach to the conclusion that the values of the dynamic modules are higher with 30÷40%. This can be explained with the occurring relaxation processes with a wide range of relaxation periods. Even in mechanical oscillations with considerable frequency, relaxation processes occur in polymer concrete composites with relaxation periods that are shorter than the oscillation period. These relaxation processes cause an aftereffect expressed in further deformation that is as smaller, as the greater is the frequency of the mechanical oscillations.

Composite no.3 has maximum values of the modules E and E_d .

5. References

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