EXPERIMENTAL DETERMINATION OF THE ELASTIC CONSTANTS OF POLYMER CONCRETE COMPOSITES

ЕКСПЕРИМЕНТАЛНО ОПРЕДЕЛЯНЕ НА ЕЛАТИЧНИТЕ КОНСТАНТИ НА ПОЛИМЕРБЕТОННИ КОМПОЗИТИ

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***Abstract**: This article has experimental nature. The values of the dynamic elastic modulus (Young's modulus) of gamma polymer concrete composites are determined quantitatively. The tested samples have the form of a rectangular parallelepiped (type bar).*

The dynamic Young's modulus of the samples is determined by the method of the experimental modal analysis. Impact excitation was applied for expressing the resonance frequencies in the tests of bending and twisting the sample. The test results were processed with regression analysis.

***Keywords**: POLYMER CONCRETE COMPOSITES, DYNAMIC MODULUS OF LINEAR DEFORMATIONS, REGRESSION ANALYSIS.*

1. Introduction

Polymer concrete composites are more frequently used as structural material for bodies and body parts of production machines, due to their good strength and deformation properties, as well as damping properties [7]. Generally until this moment, the elastic properties of these materials were studied in static tests. One of the advanced methods for determining their dynamic elastic properties is the method of the experimental modal analysis.

Compared to the static tests, the method of the experimental modal analysis [4] has a number of advantages, the most important of which are that it does not involve destruction, provides accurate results and is very mobile.

Subject of the study in this publication are gamma polymer concrete composites, developed and made in the Laboratory for testing and studying of metal cutting machines at Technical University – Sofia, Plovdiv Branch. The experimental samples have the form of rectangular parallelepiped (type bar) with dimensions 30x30x350 mm. The subject of the study of this work is the quantitative determination of the elastic modules of gamma composites. For this reason, the theory of Timoshenko [2] was applied, regarding the vibration behavior of prismatic bars, and the standard ASTM E1876-02 [1].

2. Theoretic prerequisites

➢ The dynamic Young's modulus is calculated by the following equation [1]:

$$E_d = 0.9465(mf_f^2 / b)(L^3 / t^3)T_1 \qquad (1)$$

where:

E_d - Young's modulus,;

m - weight of the bar;

b - width of the bar;

L - length of the bar;

t - thickness of the bar;

f_f - fundamental resonant frequency of bar in flexure;

T_1 - geometric correction coefficient.

The fundamental resonance frequency at bending is measured according to the standard. It is measured at the end conditions shown on Fig.1, i.e. positioned on two supports located at the two ends of the sample, at a distance of 22% of its length L [3].

If L/t > 20, then the influence of the geometric parameters of the sample is considered as minor, and T_1 may be calculated directly by the equation (2):

$$T_1 = [1.000 + 6.585(t / L)^2] \qquad (2)$$

If L/t < 20, the value of T_1 will be calculated by the following equation:

$$T_1 = 1.000 + 6.585(1 + 0.0752\mu + 0.8109\mu^2)(t / L)^2 - 0.868(t / L)^4 - \frac{8.340(1 + 0.2023\mu + 2.173\mu^2)(t / L)^4}{1.000 + 6338(1 + 0.1408\mu + 1.536\mu^2)(t / L)^2} \qquad (3)$$

As Poisson's ratio μ is an unknown value until this moment, then in calculating T_1 from the formula (3) an estimated value is assumed for μ_0. Subsequently, the ratio may be determined by the iteration procedure after calculating the dynamic shear modulus G_d, according to the algorithm shown on Fig.2.

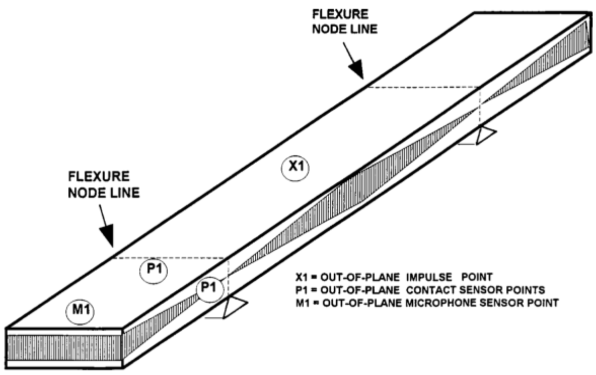

***Fig. 1.** Setting for measuring the vibrations at bending*

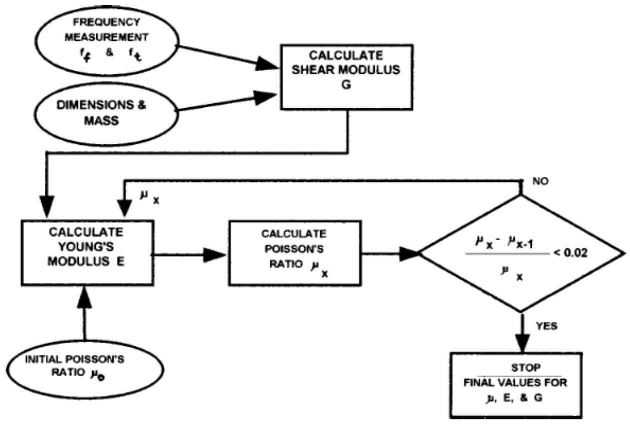

***Fig. 2.** Algorithm for determining Poisson's ratio*

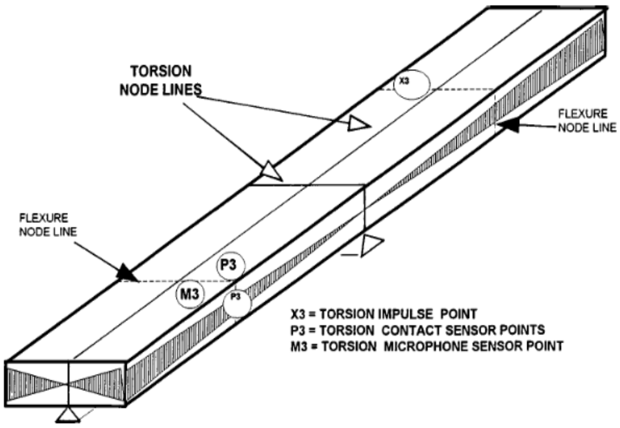

***Fig. 3.** Measurement of the vibrations of torque*

➢ The dynamic shear modulus G_d is calculated by the following equation:

$$G_d = 4(Lmf_T^2 / bt)[B/(1+A)] \quad (4)$$

where:

f_T - is the fundamental resonance torque frequency, Fig.3.

A and *B* are correction empirical coefficients for width and thickness.

$$B = \frac{b/t + t/b}{t/(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6} \quad (5)$$

$$A = \frac{[0.5062 - 0.8776(b/t) + 0.3504(b/t)^2 - 0.0078(b/t)^3]}{[12.03(b/t) + 9.892(b/t)^2} \quad (6)$$

➢ Poisson's ratio is calculated on the basis of the ratio of isotropic behavior of the material:

$$\mu = (E_d / 2G_d) - 1 \quad (7)$$

The experimental results for the dynamic modules of the linear E_d and cross G_d deformation of the polymer concrete samples are obtained by impulse excitation, Fig.4. The frequencies f_f and f_T are determined by the frequency ranges, obtained from impulse excitation of the samples, Fig.5 and 6.

The measuring equipment includes: microphone by the company "Audio-technica" - AT2031, power hammer, and specialized software "Spectra PLUS". On the grounds of the obtained reports for the frequency ranges, the data we are interested in are defined and presented in Table 1.

3. Processing of results

The experimental polymer concrete composites are multicomponent systems with interdependent and bilaterally limited components due to which the regression models represent the so called aligned (canonical) polynomials.

The mathematical and statistical processing is made with the software product MINITAB 17. For the mathematical description of the target function $\hat{y}$ - (the module of linear deformation E) an aligned model of third power of the following type is used:

$$\hat{y} = \sum_{i=1}^{q} b_i x_i + \sum_{i=1}^{q-1} \sum_{j=i+1}^{q} b_{ij} x_i x_j + \sum_{i=1}^{q-2} \sum_{j=i+1}^{q-1} \sum_{l=j+1}^{q} b_{ijl} x_i x_j x_l \quad (8)$$

Таблица 1 *Module values*

Sample №	f_f [Hz]	f_t [Hz]	*m* [kg]	G_d [Pa]	E_d [Pa]	μ
1.1	931	3243	0,738	*1,43E+10*	*3,37E+10*	*0,176*
1.2	928	3250	0,735	*1,43E+10*	*3,33E+10*	*0,163*
1.3	933	3235	0,737	*1,42E+10*	*3,38E+10*	*0,187*
2.1	872	3029	0,726	*1,23E+10*	*2,90E+10*	*0,183*
2.2	870	3021	0,725	*1,22E+10*	*2,89E+10*	*0,184*
2.3	873	3032	0,718	*1,22E+10*	*2,88E+10*	*0,183*
3.1	973	3384	0,738	*1,56E+10*	*3,68E+10*	*0,180*
3.2	966	3381	0,733	*1,54E+10*	*3,60E+10*	*0,165*
3.3	972	3378	0,731	*1,54E+10*	*3,63E+10*	*0,182*
4.1	870	3081	0,729	*1,28E+10*	*2,90E+10*	*0,138*
4.2	864	3078	0,732	*1,28E+10*	*2,87E+10*	*0,124*
4.3	873	3085	0,727	*1,28E+10*	*2,91E+10*	*0,143*
5.1	825	2908	0,701	*1,09E+10*	*2,51E+10*	*0,149*
5.2	823	2903	0,703	*1,09E+10*	*2,51E+10*	*0,147*
5.3	818	2897	0,704	*1,09E+10*	*2,48E+10*	*0,138*
6.1	730	2618	0,682	*8,62E+09*	*1,91E+10*	*0,110*
6.2	738	2637	0,676	*8,66E+09*	*1,94E+10*	*0,118*
6.3	734	2621	0,680	*8,61E+09*	*1,93E+10*	*0,119*
7.1	802	2874	0,701	*1,07E+10*	*2,37E+10*	*0,111*
7.2	796	2854	0,700	*1,05E+10*	*2,33E+10*	*0,110*
7,3	800	2868	0,694	*1,05E+10*	*2,34E+10*	*0,110*
8.1	788	2722	0,685	*9,36E+09*	*2,24E+10*	*0,196*
8.2	785	2712	0,688	*9,33E+09*	*2,23E+10*	*0,196*
8.3	792	2742	0,687	*9,52E+09*	*2,27E+10*	*0,191*
9.1	798	2838	0,708	*1,05E+10*	*2,37E+10*	*0,128*
9.2	808	2878	0,713	*1,09E+10*	*2,45E+10*	*0,125*
9.3	808	2876	0,716	*1,09E+10*	*2,46E+10*	*0,126*
10.1	832	2978	0,734	*1,20E+10*	*2,67E+10*	*0,114*
10.2	835	2980	0,728	*1,19E+10*	*2,67E+10*	*0,120*
10.3	827	2971	0,732	*1,19E+10*	*2,63E+10*	*0,106*
11.1	814	2854	0,701	*1,05E+10*	*2,44E+10*	*0,161*
11.2	811	2834	0,698	*1,03E+10*	*2,42E+10*	*0,169*
11.3	817	2864	0,701	*1,06E+10*	*2,46E+10*	*0,161*
12.1	773	2762	0,706	*9,93E+09*	*2,22E+10*	*0,118*
12.2	768	2743	0,708	*9,82E+09*	*2,20E+10*	*0,119*
12.3	772	2758	0,704	*9,87E+09*	*2,21E+10*	*0,118*
13.1	857	2957	0,717	*1,16E+10*	*2,77E+10*	*0,199*
13.2	851	2953	0,729	*1,17E+10*	*2,78E+10*	*0,185*
13.2	849	2947	0,723	*1,16E+10*	*2,74E+10*	*0,184*
14.1	938	3215	0,740	*1,41E+10*	*3,43E+10*	*0,215*
14.2	942	3217	0,745	*1,42E+10*	*3,48E+10*	*0,224*
14.3	927	3204	0,738	*1,40E+10*	*3,34E+10*	*0,195*
15.1	814	2854	0,696	*1,05E+10*	*2,43E+10*	*0,161*
15.2	809	2831	0,705	*1,04E+10*	*2,43E+10*	*0,165*
15.3	817	2850	0,693	*1,04E+10*	*2,43E+10*	*0,173*

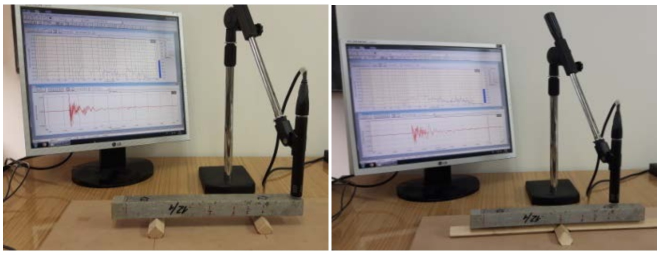

Fig. 4. *Experimental setting for determining* f_f *and* f_T

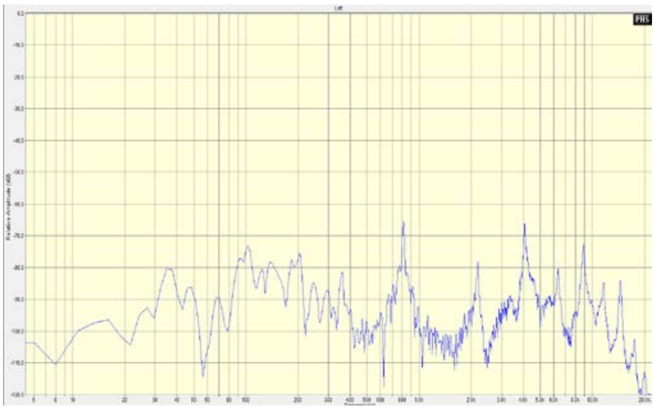

Fig.5. *Frequency range for* f_f *sample 3.2*

Fig. 6. *Frequency range for* f_T *sample 3.2*

The data in Table 1 is processed and the following regression model was obtained:

$$\hat{y} = 663x_1 + 1252x_2 - 255x_3 + 45x_4 - 9735x_1x_2 - \\ + 8509x_1x_3 - 1071x_1x_4 - 6059x_2x_3 - 1854x_2x_4 - \\ + 616x_3x_4 + 11988x_1x_2x_3 + 13391x_1x_2x_4 + \\ - 14193x_1x_3x_4 + 5012x_2x_3x_4 \quad (9)$$

Model adequacy:

The coefficient of determination is calculated, namely $R^2 = 98,96\%$, while the corrected coefficient of determination has the following value $R^2_{adj} = 98,53\%$. In order to check the significance of R^2 a value has been calculated, having Fisher's distribution:

$$F = \frac{s_R^2}{s_{rez}^2} = \frac{0,7604}{0,0033} = 227,97 \quad (10)$$

In Table [5] with Fisher's distribution F_T was calculated with significance level $\alpha = 0,05$ and degrees of freedom $v_R = 13$; $v_{rez} = 31$:

$$F_T = F(0,05;13;31) = 2,01 \quad (11)$$

As $F > F_T$ $(131,1 > 2,01)$ a conclusion can be made that R^2 is significant.

In order the coefficient to be significant, the following have to be valid $|t_i| \geq t_T(\alpha; v_{rez})$. In Table [6] with Student's distribution, with level of significance $\alpha = 0,05$ the following was reported $t_T = (0,05;31) = 1,695$. On the basis of a comparison of t_T with the values of Table [6], the conclusion can be made that almost all coefficients are insignificant with the exception of one.

The analysis of the residuals is made by means of the charts of standardized residuals, Fig.7.

Table 2 *Quote from the report*

Term	Coef	SE Coef	T	P	VIF
C2	663	99,55	*	*	2655742
C3	1252	145,07	*	*	3605541
C4	-255	339,92	*	*	1039105
C5	45	5,12	*	*	188941
C2*C3	-9735	1100,56	-8,85	0,000	4122959
C2*C4	8509	2685,61	3,17	0,003	1289979
C2*C5	-1071	151,49	-7,07	0,000	3250748
C3*C4	-6059	1132,53	-5,35	0,000	147043
C3*C5	-1854	216,49	-8,56	0,000	4166049
C4*C5	616	415,70	1,48	0,148	793799
C2*C3*C4	11988	2543,66	4,71	0,000	14755
C2*C3*C5	13391	1509,66	8,87	0,000	3982846
C2*C4*C5	-14193	3396,73	-4,18	0,000	1042078
C3*C4*C5	5012	1509,66	3,32	0,002	129269

The analysis of the residuals is made by means of the charts of standardized residuals, Fig.7.

Fig. 7 clearly shows the presence of 3 mistakes: Observation №25, №26 and №27. The experimentally measured values of the controllable parameter are: $y_{25} = 2,370$, $y_{26} = 2,450$ and $y_{27} = 2,460$, while the forecasted values of the model are: $y_{25,26,27} = 2,562$. The difference is significant and respectively the standardized residual is $d_{25} = -3,49$, $d_{26} = -2,04$ and $d_{27} = -1,85$. This provides us grounds to eliminate observations №25, №26 and №27 to process the data again.

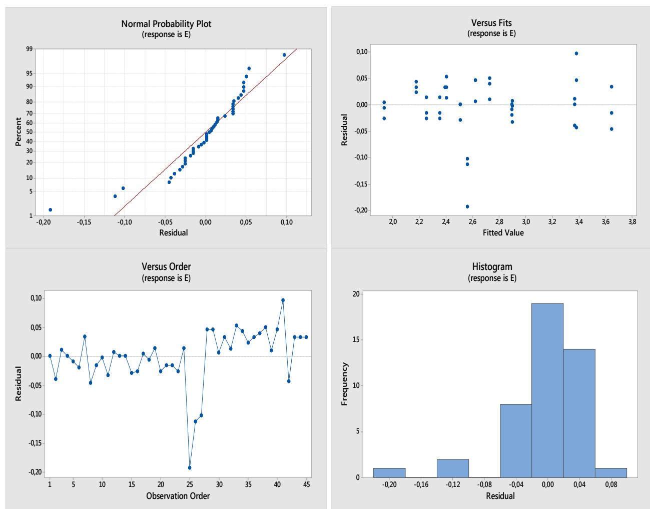

Fig. 7. *Graphs residues*

After their elimination the following regression model is obtained:

$$\hat{y} = 622x_1 + 1206x_2 - 171x_3 + 43x_4 - 9386x_1x_2 - \\ + 7784x_1x_3 - 1010x_1x_4 - 6431x_2x_3 - 1787x_2x_4 - \\ + 502x_3x_4 + 12977x_1x_2x_3 + 12951x_1x_2x_4 + \\ - 13203x_1x_3x_4 + 5451x_2x_3x_4 \quad (12)$$

Model adequacy:

The coefficient of determination is calculated $R^2 = 99,83\%$, while the corrected coefficient of determination has the following value $R^2_{adj} = 99,75\%$. In order to check the significance of R^2 a values was calculated, having Fisher's distribution:

$$F = \frac{s_R^2}{s_{rez}^2} = \frac{0,7028}{0,005} = 1222,93 \quad (13)$$

In Table [5] with Fisher's distribution F_T was calculated with level of significance $\alpha = 0,05$ and degrees of freedom $v_R = 13$; $v_{rez} = 27$:

$$F_T = F\,(0,05; 13; 27) = 2,38 \quad (14)$$

As $F > F_T$ $(131,1 > 2,38)$ a conclusion can be made that R^2 is significant.

In order the coefficient to be significant, the following have to be valid $|t_i| \geq t_T\ (\alpha; v_{rez})$. In Table [6] with Student's distribution, with level of significance $\alpha = 0,05$ the following was reported $t_T = (0,05; 27) = 1,703$. On the basis of a comparison of t_T with the values of Table [6], the conclusion can be made that almost all coefficients are insignificant with the exception of one.

The analysis of the residuals is made by means of the charts of standardized residuals, Fig.8.

Table 3 *Quote from the report*

Term	Coef	SE Coef	T	P	VIF
C2	622	41,46	*	*	2439769
C3	1206	60,79	*	*	3363953
C4	-171	147,27	*	*	1086899
C5	43	2,14	*	*	174008
C2*C3	-9386	464,55	-20,20	0,000	3907914
C2*C4	7784	1143,94	6,80	0,000	1305030
C2*C5	-1010	63,10	-16,00	0,000	2980424
C3*C4	-6431	490,13	-13,12	0,000	153795
C3*C5	-1787	90,90	-19,65	0,000	3887525
C4*C5	502	180,16	2,79	0,010	829376
C2*C3*C4	12977	1099,18	11,81	0,000	15395
C2*C3*C5	12951	641,17	20,20	0,000	3803625
C2*C4*C5	-13203	1442,64	-9,15	0,000	1045727
C3*C4*C5	5451	641,17	8,50	0,000	129817

The analysis of the residuals is made by means of the charts of the standardized residuals, Fig.8.

The analysis of the residuals does not show disruption of the prerequisites for the regression analysis. On Fig. 8 it can be seen that all residuals are within the range $\pm 0,05$. Therefore a conclusion can be made that there are no gross errors.

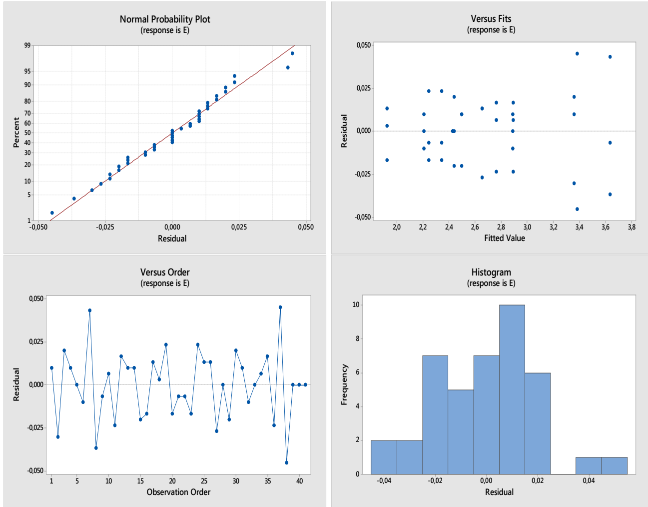

Fig. 8. *Graphs residues*

4. Conclusion

The results from the current study can be summarized as follows:

Experimental setting was created for determining the dynamic Young's modulus - E_d, the dynamic shear module - G_d, and Poisson's ratio - μ, by testing the experimental samples according to the impulse method of excitation of their dynamic system.

Regression mathematicostatistical models are deduced (by using the software product MINITAB.17), which adequately and well describe the functional relation between the target functions - elastic modulus E_d and the components of the polymenr concrete composite.

5. References

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The publication is related to work on a research project № 152PD0048-24/2015 on "Research and optimization of range polymer concrete composites in terms of their physical constants."