# THE NUMERICAL-ANALYTIC SUBSTANTIATION OF THE POSSIBILITY OF AUTOMATED MOTION CONTROL OF AN AUTONOMOUS RIGID BODY WITHOUT ITS OWN PROPULSION SYSTEM IN INCOMPRESSIBLE STRATIFIED VISCOUS FLUID

## ЧИСЛЕННО-АНАЛИТИЧЕСКОЕ ОБОСНОВАНИЕ ВОЗМОЖНОСТИ АВТОМАТИЗИРОВАННОГО УПРАВЛЕНИЯ ДВИЖЕНИЕМ АВТОНОМНОГО ТВЕРДОГО ТЕЛА БЕЗ СИЛОВОЙ УСТАНОВКИ В СТРАТИФИЦИРОВАННОЙ ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ

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**Abstract:** *The report presents a mathematical model of the motion control of an autonomous solid body moving in incompressible stratified viscous fluid and analytical and numerical analysis of this model. It is assumed that the body does not have its own propulsion system, but is equipped with controlled rudders - wings of finite span. It is moved by the influence of the buoyancy force and wings lift. The control is produced by the angle of attack of the wing change for ensuring access to the given point by this solid body. This body motion is considered to be plane-parallel motion. This paper results are based on the mathematical model which was presented by the authors at XII MTM Congress held in September 2015 and XIII MTM Congress held in March 2016.*

**KEYWORDS:** MOTION OF SOLIDS IN A FLUID, TRAFFIC COTROL, BUOYANCY FORCE, ENSURING ACCESS TO THE GIVEN *POINT, WINGS OF FINITE SPAN, WINGS LIFT*

## *1. Introduction*

The effectiveness of observations and measurements obtained in the study of the underwater world via underwater vehicles, in particular, unmanned, depends on minimizing the impact of these submersible crafts to surrounding underwater environment. First of all, it refers to a moving apparatus, which movement is carried out by various power plants (screw propeller or other propulsion). Therefore, the reduction or elimination of such effects is an important application. The ideal situation would obviously be the complete lack of engine. This means that movement control of the body can be carried out only by natural hydrodynamic forces, for instance, the Archimedes buoyancy or an wing lift effect (the body can be equipped with wing). Basic terminology and classical results for the body's motion in continuum can be found in the books [1, 2].

## *2. Accepted assumptions*

As an autonomous rigid body, the authors propose to consider a research submersible – a uniform sphere-shaped rigid body with two similar symmetrically located around the ball centre wings (fig. 1). Actually other modifications of mutual bracing of the sphere-shaped body and wings are possible. However, the proposed mathematical model can be taken as a basis for whole these alternatives.

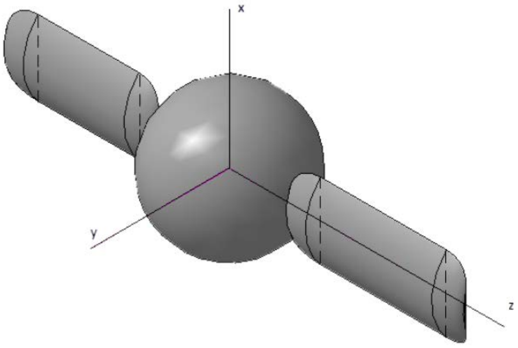

Fig. 1. Schematic submersible craft image.

The motion of submersible craft is assumed to happen in a limitless borehole bottom reservoir with an ideal incompressible non-conducting stratified liquid with viscosity effect. The viscosity is taken into account as a Stokes' drag force. It is also assumed that each layer has own density, which is known. Furthermore, liquid in each layer can move rectilinearly and uniformly with known velocity along the horizontal axis, which is perpendicular to a wingspread.

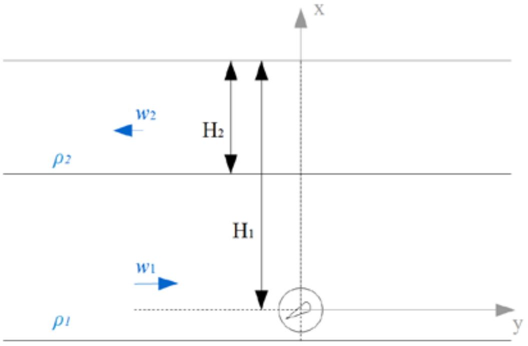

Fig. 2. Double-layer continuum figure.

In this paper the authors consider plane-parallel motion of submersible craft case. At the initial time this body is located in stationary state at a predetermined depth (fig. 2).

It is necessary to define the obtaining solution algorithm in a double-layer liquid for building a similar solution in stratified liquid.

It is usually understood that an inverse problem is a problem of control synthesis which use leads to achievement of the defined preselected value.

The goal of this research is a substantiation of existence of well-defined problem description in concerned field. There is considered to be a problem of submersible craft appearance in surface in small neighborhood of given point.

## 3. Mathematical model

At the previous authors paper [3] mathematical model of the submersible craft plane-parallel motion was constructed. It allows controlling the body through wings angle of attack modifications:

$$\begin{cases}\left(m+\frac{2}{3}\rho\pi R^3\right)\frac{d^2x}{dt^2}=F_{arch}-2F_i\cos\delta-\left(F_{drag}^{(1)}+2F_{drag}^{(2)}\right)\cos\delta-2F_{lift}\sin\delta-F_g;\\ \left(m+\frac{2}{3}\rho\pi R^3\right)\frac{d^2y}{dt^2}=-2F_i\sin\delta-\left(F_{drag}^{(1)}+2F_{drag}^{(2)}\right)\sin\delta-2F_{lift}\cos\delta.\end{cases}$$

Here $F_{arch}$ – is the buoyancy force, $F_{drag}^{(j)}=C_X^{(j)}S^{(j)}\frac{\rho v^2}{2}$ – the head resistance force for a sphere (j=1) and wings (j=2), $F_{lift}=\rho v^2 S\frac{k\alpha}{1+\mu_0}$ – the wing lift, $F_i=\frac{\rho}{2}v^2 S\frac{\mu_0}{2k}\left(\frac{2k\alpha}{1+\mu_0}\right)^2$ – the induced drag force (details can be found in [3]).

The analysis of constructed mathematical model in terms of the possibility of motion trajectory control of the submersible craft by the attack angle continuous variation is produced at this paper. A term "motion trajectory of the submersible craft" means motion trajectory of gravity center of the submersible craft (the sphere center).

As a rule, it is supposed that an attack angle is a small quantity [4]. Authors also allow this assumption. The attack angle smallness necessitates the angle $\delta$ smallness. Therefore, we can accept $\sin\delta\approx\delta, \cos\delta\approx 1$.

Using the standard change of variables:

$$x=z_1 \qquad y=z_3$$
$$\dot{x}=\dot{z_1}=z_2 \quad \dot{y}=\dot{z_3}=z_4,$$

initial mathematical model is reduced to the nonlinear differential equation system:

$$(1)\quad\begin{cases}\dot{z_1}=z_2,\\ \dot{z_2}\cdot b_0=b_1-(b_2\cdot\alpha^2+b_3+2b_4)\cdot({z_2}^2+{z_4}^2)-\\ -2b_5\cdot\alpha\cdot z_4\cdot\sqrt{{z_2}^2+{z_4}^2},\\ \dot{z_3}=z_4+w,\\ \dot{z_4}\cdot b_0=-(b_2\cdot\alpha^2+b_3+2b_4)\cdot z_4\cdot\sqrt{{z_2}^2+{z_4}^2}+\\ +2b_5\cdot\alpha\cdot({z_2}^2+{z_4}^2).\end{cases}$$

Here coefficients are defined as:

$b_0=m+\frac{2}{3}\rho\pi R^3,\quad b_1=\rho gV-mg,\quad b_2=\rho S_{kp}\frac{2k\mu_0}{(1+\mu_0)^2},\quad b_3=c_{0\_sph}\frac{\rho\pi R^2}{2}, b_4=c_{0\_w}\frac{\rho S_w}{2}, b_5=\rho S_{kp}\frac{k}{1+\mu_0}.$

A motion trajectory of the submarine craft can be modified by the wing attack angle $\alpha$ values variations. For instance, in this paper the following assumptions are considered:

1) The attack angle is able to change at a constant speed, which equals 0.5 degrees per second;
2) Attack angle threshold requirements are ±15 degrees;
3) The attack angle at the initial time equals 0.

## 4. Numerical examples

Fourth-order of accuracy Runge-Kutta method was applied for numerical solution of the differential equation system (1). Software called MATLAB 7.10.0 (R2010A) is used.

### 4.1. Homogeneous liquid

Motion of the submersible craft in homogeneous (single-layer) ideal incompressible viscous fluid with shear flow in the line of horizontal axis is considered.

Authors examine three cases of the variation law of attack angle (fig. 3):

1) $\alpha=0$;
2) $\alpha=\begin{cases}0.5\cdot t,\ if\ 0\le t\le 30,\\ 15, if\ 30<t.\end{cases}$
3) $\alpha=\begin{cases}-0.5\cdot t,\ if\ 0\le t\le 30,\\ -15, if\ 30<t.\end{cases}$

Then appropriate motion trajectories of the submarine craft can be calculated by solving the system (1) with zero initial conditions (fig. 4).

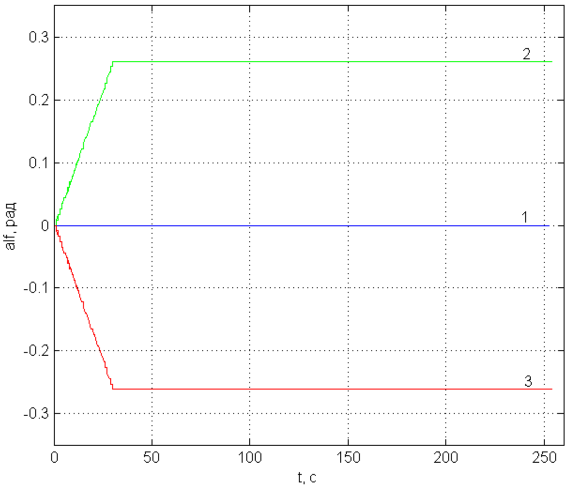

Fig. 3. Cases of the variation law of attack angle of the submarine craft in homogeneous fluid.

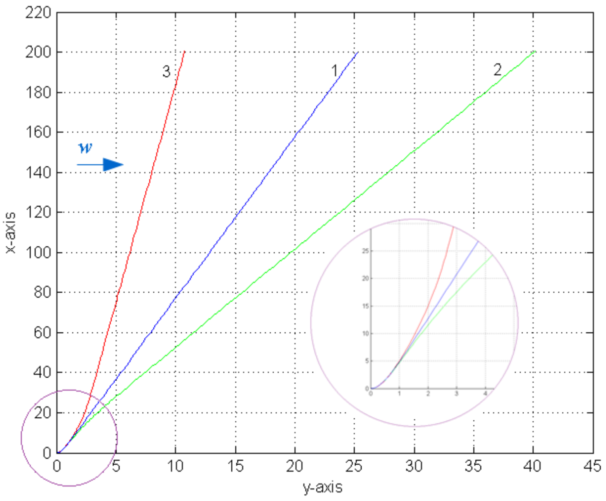

Fig. 4. Motion trajectories of the submarine craft for different cases of the variation law of attack angle in homogeneous fluid.

### 4.2. Double-layer liquid

Motion of the submersible craft in STRATIFIED (double-layer) ideal incompressible viscous fluid with shear flow in the line of horizontal axis is considered.

Authors examine three cases of the variation law of attack angle (fig. 5):

1) $\alpha=0$;
2) $\alpha=\begin{cases}0.5\cdot t,\ if\ 0\le t\le 30<\hat{t},\\ 15, if\ \ 30\le t\le\hat{t},\\ 15-0.5\cdot(t-\hat{t}), if\ \hat{t}\le t\le\hat{t}+60,\\ -15, if\ \ \hat{t}+60\le t.\end{cases}$
3) $\alpha=\begin{cases}-0.5\cdot t,\ if\ 0\le t\le 30\ <\hat{t},\\ -15, if\ 30\le\le<\hat{t},\\ -15+0.5\cdot(t-\hat{t}), if\ \ \hat{t}\le t\le\hat{t}+60,\\ 15, if\ \ \hat{t}+60\le t.\end{cases}$

Here $\hat{t}$ – is an ascent time of the submarine craft in bottom layer.

Then appropriate motion trajectories of the submarine craft can be calculated by solving the system (1) (fig. 6). The differential equation system is sequentially solved for each layer starting with the bottom layer. Its initial conditions are supposed zero conditions. For other layers initial conditions are recalculated depending on coordinates of inertia center of the submarine craft at the transitional point from layer to layer.

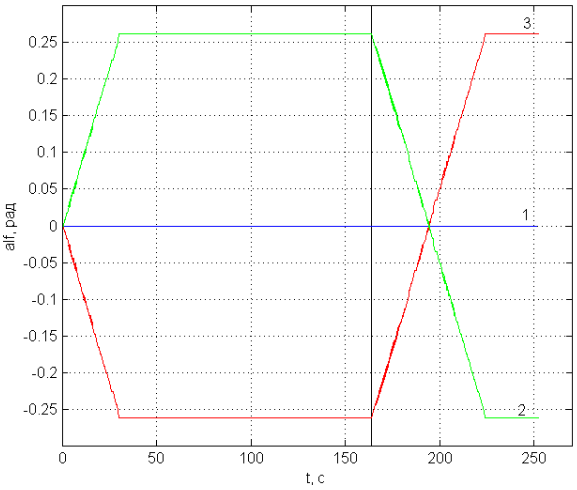

Fig. 5. Cases of the variation law of attack angle of the submarine craft in double-layer fluid.

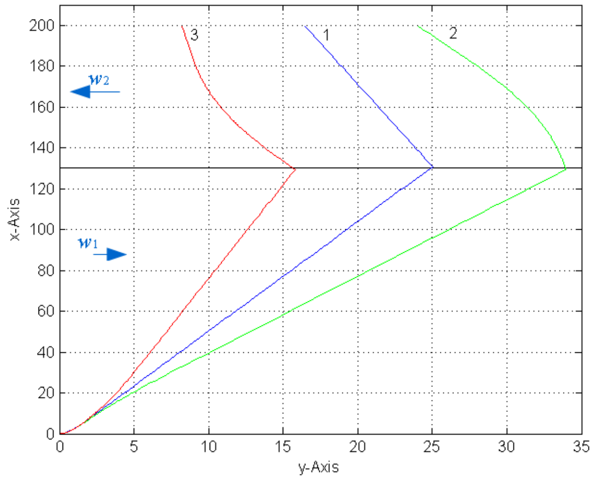

Fig. 6. Motion trajectories of the submarine craft for different cases of the variation law of attack angle in double-layer fluid.

At these examples, the following values of quantities are offered. The diameter of the surfaced body (ball) is 1 meter; its mass is calculated like $m = 0.98\rho V$, where $\rho$ is averaged body density. It is assumption to consider rectangular wings with wingspan 1 meter, aspect ratio of the wing 5 and relative maximum thickness 16 %. An initial immersion depth $H_1$ equals 200 meters. In case of homogeneous liquid its density is supposed to be 1038 kg/m$^3$, shear flow velocity – $|\vec{w}| = 0.1$ m/s. shear flow velocity. The second layer depth H2 is 70 meters. Liquid densities in different layers equals $\rho_1$= 1050 kg/m$^3$ and $\rho_2$=1025 kg/m$^3$. Shear flows have velocities $|\overrightarrow{w_1}| = 0.15$ m/s and $|\overrightarrow{w_2}| = 0.1$ m/s.

## 5. Conclusion

At this paper the motion trajectory of the submarine craft calculation algorithm is presented. Mentioned body can rise up under the influence of the Archimedes buoyancy and a wing lift effect.

The continuously varying wing attack angle control possibility of the submarine craft movement is analyzed. Besides that neighbourhood, for each point of which can be set an inverse well-defined problem, is obtained.

## 6. Literature

1. Vallander S.V. Lekcii po gidroaeromehanike [Lectures on hydrodynamics]. – St. Petersburg, St. Petersburg Univ. Press, 2005. 304 p. (In Russian)
2. Kochin N.E., Kibel' I.A., Roze N.V. Teoreticheskaja gidromehanika [Theoretical Hydromechanics]. Moskow, Fizmatlit Publ., 1963. Vol. 1-2. (In Russian)
3. Firsov A. N., Kuznetsova L. V., Sorokina N. V. The solution of the inverse problem of motion control of a rigid body, pop-up in a stratified incompressible viscous fluid under the influence of the Archimedes force // XII International Congress “Machines, Technologies, Materials” (Varna, Bulgaria, 16-19.09.2015). Proceedings, Vol. 3. – P. 3-7. (2015)
4. Martinov A. K. Experimental Aerodynamics. Moskow, Oborongiz, 1950. 475 p. (In Russian)