THE PROBLEM OF OVERLAPPING PROJECT ACTIVITIES WITH INTERDEPENDENCY

Prof. Gurevich G., Prof. Keren B., Prof. Laslo Z.
Department of Industrial Engineering and Management – SCE-Shamoon College of Engineering, Beer Sheva, Israel

gregoryg@sce.ac.il, baruchke@sce.ac.il, zohar@sce.ac.il

Abstract: *This paper analyzes a simple project with two activities. The activities can be executed in a serial mode (separately), in a parallel mode (overlapping), or in a mixed mode (partly in a serial mode and partly in a parallel mode). There is interdependence between the activities and the duration-budget tradeoffs functions of the activities are defined differently for each execution strategy. The paper presents a deterministic duration-budget tradeoff model that takes into account the interdependence between the activities in order to determine the optimal execution-mode and the budget distributions among the project activities. A stochastic extension of the proposed model is also considered. The presented analysis can help project managers and practitioners in choosing the optimal overlapping strategy for different objective functions and constraints.*

Keywords: DURATION-BUDGET TRADEOFF, OVERLAPPING, INTERDEPEBDENCE, TIME CONSTRAINT.

1. Introduction

There are three main techniques for project time compression: activity crashing, substitution and overlapping (Hazini et al. [1]). Crashing is a schedule compression technique in which cost and schedule tradeoffs are analyzed to determine how to optimize the best schedule compression for the least cost (Laslo and Gurevich [2]-[5], Laslo et al. [6]). Activity substitution involves the replacement of one activity or a sequence of activities in a series by other activities. Overlapping is a schedule compression technique in which activities that normally executed in sequence, are executed in parallel. Terwiesch and Loch [7] claimed that overlapping research and development activities are widely used to reduce project completion times. Dehghan et al. [8] restated that for a typical construction project, a number of overlapping strategies exist which can result a timesaving. The cost of these strategies varies significantly depending on the total rework and complexity they generate. Therefore, the best overlapping strategy that generates the required timesaving at a minimum cost should be found.

This paper analyzes a simple project with two activities. The activities can be executed separately, with overlapping (in a parallel mode), or in a mixed mode (partly separately and partly in a parallel mode). There is interdependence between the activities and the duration-budget tradeoff functions of the activities are defined differently if the activities are executed separately, in a parallel mode or in a mixed mode. The presented deterministic duration-budget tradeoff model takes into account the interdependence between the activities and helps to determine the optimal strategy for different objective functions and constraints. In particular, the model allows to determine: 1) the optimal execution-mode that minimizes the total project duration given that the budget of each activity is already allocated; 2) the optimal execution-mode and the budget distribution that minimize the total project duration, given a fixed total budget that should be optimally distributed among the activities; 3) the optimal execution-mode, the budget size and the budget distribution among the activities that minimize the total expenses, given the penalties/bonuses for a delay/early completion time; 4) the optimal execution-mode and the budget distribution that minimize the total budget for the project, given a constraint on the completion time of the project. A stochastic extension of the proposed model is also being considered. In the stochastic context, the activities durations are random variables where the expected values of the durations are results of the budget size that is invested in each activity

2. Model description

The *duration-budget tradeoff* functions for activities $i=1,2$ are defined differently for situations where the activities executed separately or jointly.

If the two activities are executed separately, then the *duration-budget tradeoff* functions for the first and second activities are defined by the following equation

$$
\begin{aligned}
t_1 &= f_1(b_1),\\
t_2 &= f_2(b_2),
\end{aligned}
\tag{1}
$$

where, t_i is the i's activity duration if both activities are executed in series, b_i is the budget allocated to activity i, $b_{iN} \le b_i \le b_{iC}$, b_{iN} is the known normal (minimal) budget that can be allocated to activity i, b_{iC} is the known crash (maximal) budget that can be allocated to the activity, $f_i(b_i)$ is the estimated (known) *duration-budget tradeoff*, a decreasing function that characterizes the duration of the i's activity when both activities are executed in series. The end points of the function $f_i(b_i)$ are the normal (maximal) duration $f_i(b_{iN})$ corresponding to the known normal (minimal) budget b_{iN}, and the crash (minimal) duration, $f_i(b_{iC})$ corresponding to the known crash (maximal) budget b_{iC}, $f_i(b_{iC}) \le f_i(b_i) \le f_i(b_{iN})$, $i=1,2$.

If the two activities are executed in parallel, then the *duration-budget tradeoff* functions for the first and second activities are defined by the following equation

$$
\begin{aligned}
t_1^* &= f_1^*(b_1),\\
t_2^* &= f_2^*(b_2),
\end{aligned}
\tag{2}
$$

where, t_i^* is the i's activity duration if the two activities are executed in parallel, $f_i^*(b_i)$ is the estimated (known) *duration-budget tradeoff*, a decreasing function that characterizes the duration of the i's activity if both activities are executed jointly. The end points of the *duration-budget tradeoff* function $f_i^*(b_i)$ are the normal (maximal) duration $f_i^*(b_{iN})$ corresponding to the known normal (minimal) budget b_{iN}, and the crash (minimal) duration, $f_i^*(b_{iC})$ corresponding to the known crash (maximal) budget b_{iC}, $f_i^*(b_{iC}) \le f_i^*(b_i) \le f_i^*(b_{iN})$, $i=1,2$.

The model assumes that the rate of execution of both activities is constant. Let x and y be decision variables that defined the portions of the first and the second activity, respectively, that are executed jointly, $0 \le x = x(b_1, b_2) \le 1$, $0 \le y = y(b_1, b_2) \le 1$. Then, by (1)-(2), the total durations of the two activities are:

$$xt_1^* + (1-x)t_1 = xf_1^*(b_1) + (1-x) f_1(b_1), \qquad (3)$$
$$yt_2^* + (1-y)t_2 = yf_2^*(b_2) + (1-y) f_2(b_2),$$

respectively. Moreover, since $xf_1^*(b_1) = yf_2^*(b_2)$, then by equation (3) the total duration of the project is

$$t = xf_1^*(b_1) + (1-x) f_1(b_1) + \left(1 - x\frac{f_1^*(b_1)}{f_2^*(b_2)}\right) f_2(b_2). \qquad (4)$$

3 Analysis of working strategies and budget distributions among the project activities

This section analyses the optimal working strategies and budget distributions among the project activities in the context of four different objective functions.

3.1. The optimal strategy that minimizes the total project duration given a fixed budget allocated to each activity

The considered problem is to minimize the total duration of the project, t, given the values of b_1 and b_2, $b_{1N} \le b_1 \le b_{1C}$, $b_{2N} \le b_2 \le b_{2C}$. It means that the problem is to determine the value $x = x^*$ that minimizes the total duration of the project. Since $0 \le x \le 1$, $0 \le y \le 1$, then the optimal portion of overlapping x^* can be expressed by equation (4) as follows

$$x^* = \underset{0 \le x \le min\left\{1, \frac{f_2^*(b_2)}{f_1^*(b_1)}\right\}}{arg\ min} \left\{ \begin{array}{l} xf_1^*(b_1) + (1-x) f_1(b_1) \\ + \left(1 - x\frac{f_1^*(b_1)}{f_2^*(b_2)}\right) f_2(b_2) \end{array} \right\}. \qquad (5)$$

The following Proposition 1 presents the solutions x^* of equation (5).

Proposition 1. Given values of b_1 and b_2 ($b_{1N} \le b_1 \le b_{1C}$, $b_{2N} \le b_2 \le b_{2C}$), the solutions x^* of equation (5) are defined as follows.

If $f_1^*(b_1)\left(1 - \frac{f_2(b_2)}{f_2^*(b_2)}\right) - f_1(b_1) > 0$ then equation (5) has the unique solution $x^* = 0$,

If $f_1^*(b_1)\left(1 - \frac{f_2(b_2)}{f_2^*(b_2)}\right) - f_1(b_1) < 0$ then equation (5) has the unique solution

$$x^* = min\left\{1, \frac{f_2^*(b_2)}{f_1^*(b_1)}\right\} = \begin{cases} 1 & \text{if } f_2^*(b_2) \ge f_1^*(b_1) \\ \frac{f_2^*(b_2)}{f_1^*(b_1)} & \text{if } f_2^*(b_2) < f_1^*(b_1) \end{cases},$$

If $f_1^*(b_1)\left(1 - \frac{f_2(b_2)}{f_2^*(b_2)}\right) - f_1(b_1) = 0$ then each $x^* \in \left[0, min\left\{1, \frac{f_2^*(b_2)}{f_1^*(b_1)}\right\}\right]$ is a solution of equation (5).

3.2. The optimal strategy and budget distribution that minimize the total project duration given a fixed total budget for the project

Let b be an additional available budget that can be added to the normal (minimal) budget of the project's activities, $b_{1N} + b_{2N}$, $0 \le b \le (b_{1C} + b_{2C}) - (b_{1N} + b_{2N})$, and b_{ai} be an additional budget that can be added to the normal (minimal) budget of the i's project activity $0 \le b_{ai} \le b_{iC} - b_{iN}$, $i = 1,2$. That is, the budgets that can be allocated to the first and second project activities are $b_1 = b_{1N} + b_{a1}$, $b_2 = b_{2N} + b_{a2}$, respectively, where $b_{a2} = b - b_{a1}$. The considered problem is to minimize the total project duration, t, given the amount b of the additional budget. In other words, the problem is to determine values $b_{a1} = b_{a1}^*$ and $x = x^*$ of equation (4) such that

$$\left(b_{a1}^*, x^*\right) = \underset{(b_{a1}, x) \in A}{arg\ min} \left(\begin{array}{l} xf_1^*(b_{1N} + b_{a1}) + (1-x) f_1(b_{1N} + b_{a1}) \\ + \left(1 - x\frac{f_1^*(b_{1N} + b_{a1})}{f_2^*(b_{2N} + b - b_{a1})}\right) f_2(b_{2N} + b - b_{a1}) \end{array} \right), \qquad (6)$$

$$A = \left\{ \begin{array}{l} (b_{a1}, x) : 0 \le b_{a1} \le min\{b, b_{1C} - b_{1N}\}, \\ 0 \le x \le min\left\{1, \frac{f_2^*(b_2)}{f_1^*(b_1)}\right\} \end{array} \right\}.$$

Equation (6) is a standard mathematical programing problem of minimization of a function with two decision variables, b_{a1} and x.

3.3. The optimal strategy and the budget distribution that minimize the total project expenses given the penalties for a delay and the bonuses for earlier completion time

Let t_0 be a given agreed completion time of the project. The known function $g(t - t_0)$ defines penalties/bonuses for a delayed/ earlier completion time. That is, $g(t - t_0) \ge 0$ if $t \ge t_0$, and $g(t - t_0) \le 0$ if $t \le t_0$. The total project expenses are defined as

$$c = b_1 + b_2 + g(t - t_0), \tag{7}$$

where $b_{iN} \le b_i \le b_{iC}$, $i = 1,2$, $t = t(b_1, b_2, x)$ are defined by equation (4). The considered problem is to minimize the total project expenses c as defined by equation (7). Thus, the problem is to determine the values $b_i = b_i^*$, $i = 1,2$, and $x = x^*$ such that

$$\left(b_1^*, b_2^*, x^*\right) = \underset{(b_1, b_2, x) \in B}{arg\ min} \left\{b_1 + b_2 + g(t - t_0)\right\}, \tag{8}$$

$$B = \left\{ \begin{matrix} (b_1, b_2, x) : b_{1N} \le b_1 \le b_{1C}, b_{2N} \le b_2 \le b_{2C}, \\ 0 \le x \le min\left\{1, \dfrac{f_2^*(b_2)}{f_1^*(b_1)}\right\} \end{matrix} \right\}.$$

Equation (8) is a standard mathematical programing problem of minimization of a function with three decision variables: b_1, b_1 and x ($b_{iN} \le b_i \le b_{iC}$, $i = 1,2$, $0 \le x \le min\left\{1, f_2^*(b_2) / f_1^*(b_1)\right\}$).

3.4. The optimal strategy and budget distribution that minimize the total needed budget given a time constraint on the total duration

Assume a situation where there is a time constraint on the total duration of the project, $t \le t_0$, where t_0 is a known value. The considered problem is to minimize the total budget, $b_1 + b_2$, subject to a given time constraint on the total duration, $t \le t_0$, where t is defined by equation (4). Thus, the problem is to determine the values $b_i = b_i^*$, $i = 1,2$, and $x = x^*$ such that

$$\begin{aligned} &\left(b_1^*, b_2^*, x^*\right) = \underset{(b_1, b_2, x) \in B}{arg\ min} \left\{b_1 + b_2\right\} \\ &s.t. \\ &t \le t_0, \end{aligned} \tag{9}$$

where B is defined in equation (8). This is also a standard mathematical programming problem.

4. A stochastic extension of the model

In this section, the durations of the activities $i = 1,2$ are assumed to be random variables. Similarly to the model presented in section 3, the *duration-budget tradeoff* functions for activities $i = 1,2$ are defined differently when the activities are executed in a serial mode or in a parallel mode.

If the two activities are executed separately, then the *duration-budget tradeoff* functions for the first and the second activities are defined by the following equation

$$\begin{aligned} t_1 &= f_1(b_1) + \varepsilon_1(b_1), \\ t_2 &= f_2(b_2) + \varepsilon_2(b_2) \ . \end{aligned} \tag{10}$$

where, b_i, $f_i(b_i)$ are defined as in equation (1), $i = 1,2$, $\varepsilon_1(b_1)$ and $\varepsilon_2(b_2)$ are independent random variables with zero expectations, $E(\varepsilon_1(b_1)) = E(\varepsilon_2(b_2)) = 0$.

If the two activities are executed jointly, then the *duration-budget tradeoff* functions for the first and the second activities are defined by the following equation

$$\begin{aligned} t_1^* &= f_1^*(b_1) + \varepsilon_1^*(b_1), \\ t_2^* &= f_2^*(b_2) + \varepsilon_2^*(b_2) \ . \end{aligned} \tag{11}$$

where, $f_i^*(b_i)$ is defined as in equation (2), $i = 1,2$, $\varepsilon_1^*(b_1)$ and $\varepsilon_2^*(b_2)$ are independent random variables with zero expectations, $E\left(\varepsilon_1^*(b_1)\right) = E\left(\varepsilon_2^*(b_2)\right) = 0$.

The strategy for project execution in the stochastic case can be defined as follows. The two activities should be executed in parallel until a portion x of the first activity or a portion y of the second activity will be completed, where the variables $0 \le x = x(b_1, b_2) \le 1$ and $0 \le y = y(b_1, b_2) \le 1$ are decision variables. The rest of the activities should be executed in a serial mode. The rate of execution of both of activities is assumed to be constant. In this framework, the duration where both activities are executed in parallel, $min\left\{xt_1^*, yt_2^*\right\}$, and the total duration of the project, t, are random variables. Straightforwardly, by the law of total expectation, it can be shown that the expected value of the total duration of the project is

$$\begin{aligned} &E(t) \\ &= E\left(\begin{matrix} \left(\begin{matrix} E\left(xt_1^* \middle| xt_1^* \le yt_2^*, t_2^*\right) + (1 - x)E(t_1) \\ + \left(1 - \dfrac{1}{t_2^*} E\left(xt_1^* \middle| xt_1^* \le yt_2^*, t_2^*\right)\right) E(t_2) \end{matrix} \right) \\ \times P\left(xt_1^* \le yt_2^* \middle| t_2^*\right) \end{matrix} \right) \\ &+ E\left(\begin{matrix} \left(\begin{matrix} yt_2^* + (1 - y)E(t_2) \\ + \left(1 - yt_2^* E\left(\dfrac{1}{t_1^*} \middle| xt_1^* > yt_2^*, t_2^*\right)\right) E(t_1) \end{matrix} \right) \\ \times P\left(xt_1^* > yt_2^* \middle| t_2^*\right) \end{matrix} \right) \end{aligned} \tag{12}$$

In a stochastic framework, equation (12) will be used instead of equation (1). Thus, the optimal strategies and the budget distribution among the project activities for the stochastic case can be analyzed in a similar way as was presented in section 3 for different objective functions.

5. Conclusions

In a real life projects have interdependency among their activities, and a parallel execution-mode may increase or decrease the activity duration and the total completion time of a project. We believe that the proposed analysis will help practitioners to select the optimal execution-mode, which is not trivial in the general case and can be a mixed one. Future research is needed to solve the stochastic problems for different distribution functions of project activities durations and to generalize the presented analysis for large projects with many activities.

6. References

1. Hazini, K., Dehghan, R. and Ruwanpura, J.Y. (2013). A heuristic method to determine optimum degree of activity accelerating and overlapping in schedule compression. *Canadian Journal of Civil Engineering*, 40(4), pp: 382-391.

2. Laslo, Z. and Gurevich, G. (2015). Planning and controlling projects under uncertainty. *American Journal of Operational Research*, 5(3), pp: 47-56.

3. Laslo, Z. and Gurevich, G. (2014). Enhancing project on time within budget performance by implementing proper control routines. *Management*, 72, pp: 53-69.

4. Laslo, Z. and Gurevich, G. (2013). PERT-type projects: time-cost tradeoffs under uncertainty. *Simulation: Transactions of the Society for Modeling and Simulation International*, 89(3), pp: 278-293.

5. Laslo, Z. and Gurevich, G. (2007). Minimal budget for activities chain with chance constrained lead-time. *International Journal of Production Economics*, 107(1), pp: 164-172.

6. Laslo, Z., Gurevich, G. and Keren, B. (2009). Economic distribution of budget among producers for fulfilling orders under delivery chance constraints. *International Journal of Production Economics*, 122(2), pp: 656-662.

7. Terwiesch, C. and Loch, C.H. (1999). Measuring the effectiveness of overlapping development activities. *Management science*, 45(4), pp: 455-465.

8. Dehghan, R., Hazini, K. and Ruwanpura, J. (2015). Optimization of overlapping activities in the design phase of construction projects. *Automation in Construction*, 59, pp: 81-95.